RICH MATHEMATICS TASKS, STUDENT MISCONCEPTIONS, USING TASKS

PROMPTS WITH PURPOSE: USING HIGH-QUALITY TASKS

The quality of our mathematics tasks directly affects the learning of our students (Stein & Lane, 1996). For years, there has been a belief that the quantity of mathematics trumps the quality of mathematics. And so, it was likely believed that a practice sheet with 20 or more problems was a more effective way to learn mathematics than one or two high-quality prompts or tasks.

This notion of quantity creates a myriad of problems. For one, a student who practices a skill or concept incorrectly over and over again ingrains a misconception that can be extremely difficult for us to correct. Copious amounts of low-level practice become mundane and can cause students to fall out of love with mathematics. Often, these low-level tasks don't further one's learning. They don't always, if ever, promote reasoning. The procedural focus on isolated concepts may limit our students' ability to transfer mathematical ideas to new situations. Most important, low-level tasks don't provide opportunities to engage in mathematics in the same ways we encounter it in day-to-day life. In other words, mathematics in the real world isn't scripted. We don't continue to do or use the same skill over and over in a short amount of time. In the real world, mathematics isn't isolated and certainly isn't contrived.

Selection of high-quality mathematics tasks is a foundational part of exemplary mathematics instruction. After selecting tasks, we plan for our students to work with partners or small groups. Then, we must anticipate what will happen. We must consider the questions we will ask. We must think about how we will facilitate meaningful discourse and close the lesson. All of this builds from and with conceptual understanding and mathematics vocabulary. Simply, quality mathematics tasks alone won't produce proficient students. However, proficiency can't be developed without quality tasks.

Tools of the Trade: Qualities of High-Quality Tasks

We might think of mathematics tasks as tools of our trade. Like tools designed for other jobs, we want the highest quality. We look for precision, craftsmanship, effectiveness, and practicality. The idea of high-quality mathematics tasks means different things to different people. There are all sorts

of tools and rubrics for identifying a quality mathematics task. These tools usually identify that high-quality mathematics tasks

- align to mathematics content standards and/or significant mathematical ideas.
- make use of representations.
- provide students with opportunities for communicating their reasoning.
- can be modified for multiple entry points.
- create opportunities for different strategies for finding solutions.
- allow students to make connections between concepts.
- require cognitive effort.
- are problem based, authentic, or interesting.

Selecting High-Quality Tasks

We can find mathematics tasks in textbooks, supplemental resources, and of course online. But how do we know if they are high quality? A rating or review tool can help us develop our "high-quality filter" for selecting good tasks. We can apply it to those tasks we find in print resources and online. It is important to keep in mind that there is no perfect task. Every task can be improved. This tool aligns to the characteristics of high-quality tasks described here.

Identifying High-Quality Tasks

The purpose of the task is to teach or assess:

☐ Conceptual understanding	☐ Procedural skill and fluency	☐ Application

Rating Scale:

2 - Fully Meets the Characteristic

1 - Partially Meets the Characteristic

0 - Does Not Meet the Characteristic

The mathematics task	Rating
Aligns to mathematics content standards I am teaching.	
Encourages my students to use representations.	
Provides my students with an opportunity for communicating their reasoning.	
Has multiple entry points.	
Allows for different strategies for finding solutions.	
Makes connections between mathematical concepts, between concepts and procedures, or between concepts, procedures, and application.	
Prompts cognitive effort.	
Is problem-based, authentic, or interesting.	

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The Purpose of the Task

Mathematical rigor promotes instructional balance between concepts, procedures, and applications of mathematics. So our initial consideration for selecting a task is to determine the purpose of the task. How does it connect with one of these components of rigor? Does the task engage students in concepts? Does it build procedural fluency? Does it apply concepts and procedures to problems and real-world contexts? There are other important considerations to determine the quality of the task.

The task aligns to the mathematics standards I am teaching.

Tasks must be worthwhile and aligned to the skills and concepts in our curriculum.

Tasks that fully meet this characteristic align directly to standards in my curriculum.	Tasks that partially meet this characteristic align to a standard in an adjacent grade level but are important and necessary.	Tasks that do not meet this characteristic do not connect with standards in my curriculum.

The task encourages my students to use representations.

Representations help students make sense of and communicate mathematical ideas.

Tasks that fully meet this characteristic explicitly direct students to use representations.	Tasks that partially meet this characteristic imply or provide space for representations.	Tasks that do not meet this characteristic are clearly procedural with no reference or space for representations.

The task provides my students with an opportunity for communicating their reasoning.

Students can communicate their reasoning with models or pictures, numbers, and words.

Tasks that fully meet this characteristic explicitly direct students to communicate their reasoning.	Tasks that partially meet this characteristic imply that students should communicate their reasoning.	Tasks that do not meet this characteristic do not require students to explain or justify their thinking.

The task has multiple entry points.

Students can approach a problem from various perspectives, using diverse strategies and/or representations.

Tasks that fully meet this characteristic are open to many possible solution paths and representations.	Tasks that partially meet this characteristic can be approached in different ways but may provide an example or prompt to direct students to an approach.	Tasks that do not meet this characteristic have a specific solution path intended or directed.

The task allows for different strategies for finding solutions.

Students can solve a problem in various ways.

Tasks that fully meet this characteristic are open to any strategy regardless of the efficiency of the strategy.	Tasks that partially meet this characteristic can be approached in different ways but imply a specific strategy for students to use.	Tasks that do not meet this characteristic direct students to a specific solution path or calculation.

The task makes connections between mathematical concepts.

Mathematics ideas are related. We can also connect them to representations, procedures, and applications.

Tasks that fully meet this characteristic connect mathematical ideas or connect concepts/procedures/applications within a topic.	Tasks that partially meet this characteristic allow for connections but do not call for them directly.	Tasks that do not meet this characteristic make no connections. They focus on a single procedure or recall.

Task prompts cognitive effort.

High-quality tasks should generate some amount of struggle. Students should have to make sense of the prompt, the problem, or the representation.

Tasks that fully meet this characteristic offer no obvious solution path. Or they require concepts and procedures to be applied to new situations or contexts.	Tasks that partially meet this characteristic are problem based but indicate how they can be solved.	Tasks that do not meet this characteristic provide no cognitive resistance. Students are directed to do something exact or recall a skill or concept.

Tasks are problem based, authentic, or interesting.

High-quality tasks are problem based. They can reflect real-world, authentic applications of mathematics. They should have interesting or novel prompts that grab students' attention.

Tasks that fully meet this characteristic are problem based and authentic or interesting.	Tasks that partially meet this characteristic are problem based.	Tasks that do not meet this characteristic are not problem based.

Hazards of Low-Level Tasks

Low-level tasks are typically grounded in recall. They might only require procedure for completion. They lack the characteristics that engage students in reasoning about mathematical ideas. They don't provide opportunity for discussion. These tasks present mathematics concepts in isolated fashions.

Low-level tasks create other challenges that are difficult for us to overcome. Most important, these tasks provide a limited, if any, window into student understanding. This happens because they generally require a single, correct answer. They require a specific strategy or process for successful completion. Because of this, we aren't always able to see our students' strategies, partial understanding, or misconceptions. Without this understanding, we are left to make guesses about what to do next. We are unable to directly address specific mathematics needs. We are left to reteach or provide interventions that don't necessarily address the problem.

Low-level tasks may also yield correct answers with flawed or incomplete understanding of the mathematics. A student may find the product of 5×6 by skip-counting by 5. He may apply this strategy to every multiplication expression he encounters. It may be his only strategy. If we consistently use low-level, basic multiplication prompts, we may only recognize his correct answers without learning of his only strategy. In time, his limited strategy will create considerable challenges because of the difficulty of efficiently and accurately skip-counting with larger, multi-digit numbers.

Low-level tasks

- make use of simple recall or procedure,
- require one answer that is found with a specific strategy or pathway,
- do not feature opportunities for representing mathematics,
- do not prompt for reasoning and justification, and
- lack connections within and between mathematics concepts.

Is It REALLY a Good Task?

Some tasks are quite easy to identify as low level. Typically, these tasks are pure computation. But some tasks can be quite misleading. We must keep in mind that a task that makes use of representations does not automatically qualify as a quality task. Consider prompts that ask students to identify a fractional piece of a rectangle or circle. Sure, this presents the concept with a representation. But it is simply recall of what a fraction is. In other situations, we may see a representation that is different from those we encountered as students. The following task is an example. Many of us did not shade grids to represent decimals. This "new" representation may mislead us to believe that the task is of high quality.

Shade the grid to show 0.4.
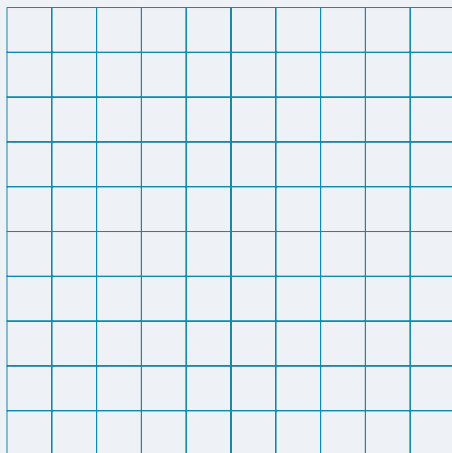
Another faulty indicator of quality is relying on tasks that require more than one right answer. Consider the following prompt.

Identify all of the numbers below that are greater than 48.

 A. 19

 B. 37

 C. 49

 D. 60

 E. 25

 F. 100

The prompt above does ask for more than one right answer. However, the cognitive demand required to complete the task is quite low. We should look for tasks that have more than one strategy or solution path rather than more than one correct answer.

Context can also mislead us about the quality of a task. The purpose of learning mathematics is to apply it to the problems we face in the real world. So it makes sense that high-quality tasks have a real-world connection. But when considering a task, we need to keep in mind if the real-world application makes sense. Is it contrived? Is it possible? Is the problem worth solving? Consider this problem:

32 ice skaters broke their arms at a figure skating competition. A hospital uses $5\frac{1}{2}$ feet of fabric to make a cast. The hospital has 300 yards of fabric available. Does the hospital have enough fabric for all of the skaters?

The task certainly provokes questions. Our students might ask how many people were in the competition? Did their parents have to sign a waiver? Why does every cast get the same amount of fabric when each person's arm is different? Who measures out the fabric exactly like that? Why did they all go to the same hospital? Yet, none of these questions are about the mathematics. Why should they be? The problem is a silly fabrication so that fractions, measurement, and multi-step problem solving can be "applied" to an authentic situation. It reminds us of the classic "two trains leaving Chicago" problem.

Some believe that manipulatives and tools are a "must-have" to determine the quality of a task. Others believe that the use of manipulatives and other tools, including calculators, cheapens or lessens the quality of a mathematics task. Neither is true. The quality of a task is determined by what you do with tools or representations rather than if you have access to them. It is important for us to remember that meaning is in the mathematics, not the manipulative.

USING QUALITY TASKS

We can use the quality tasks we select in different ways. We might choose to use them as our instructional centerpiece during a lesson. When doing this, we must

consider if students will engage in the task independently or cooperatively. We must think about what our students might do and why they might do it. We also have to consider how we will debrief the task and facilitate discussion about the task, the solution, and strategies for solutions.

We can also use these tasks for formative or summative assessment. When assessing with the tasks, we should reflect on what will satisfy the prompt. We should think about what ideas, representations, or strategies will count as evidence of understanding. We also want to begin to think about how we will use the information for our instructional next steps.

Using Tasks Instructionally

Use of quality tasks for instruction, from selection to implementation, must be intentional. An effective way to implement these tasks is through three stages.

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Launch → Engage → Debrief →
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During the first stage, we set the context for the problem, revisit skills or concepts that students might use with the task, and convey our expectations for quality work and collaboration.

In the second stage, students engage collaboratively with partners or small groups to exchange ideas, apply strategies, and adjust thinking. During the second stage, we circulate to monitor student thinking. We ask questions to focus student thinking and make note of work that we want to highlight during the debrief. At this time, we might begin to think about the sequence of student sharing. We can give students numbers written on sticky notes to help organize our sequence. As we sequence their work, we want to consider how their relationships, representations, strategies, misconceptions, and errors are connected. We want to sequence so that each new sharing builds from or contrasts with the previous idea.

During the third stage, we facilitate a discussion or gallery walk so that groups can share their solutions and strategies. In this stage, students construct their meaning of the mathematics. They share their thinking and push back on the thinking of others. At this time, we facilitate a discussion, being careful to avoid influencing or even contaminating student thoughts by offering or dismissing specific strategies that we would use or prefer.

In some situations, we might modify the approach so that students engage with tasks independently before sharing ideas with partners and eventually the whole group. In this sequence, students work with the task independently, they share their ideas with a partner, and then the whole class comes together to share strategies and insights.

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Independent Student Work → Partners Discuss → Group Shares and Teacher Charts →
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Anticipating Student Responses

We must anticipate what our students' responses may be so that our tasks, questions, and discussion will be most effective. It is impossible to anticipate every strategy or misconception that our students will have. We can't expect to know every mistake they will make. Even so, anticipating possibilities prepares us for the next instructional steps we might take during the discussion, later in the lesson, or the next day. We can develop our ability to anticipate student responses by working with other teachers to select and plan tasks. We can work to complete the problem in as many different ways as possible. We can even attempt to apply the misconceptions we think our students might have. Anticipating student thinking is highlighted throughout this book.

Misconceptions

Anticipating student work enables us to imagine the misconceptions. Misconceptions are not random. They happen when students apply faulty logic. Often, misconceptions can be explained. They occur when students make improper connections between skills or concepts. Sometimes this happens as students look to find patterns and connect them to seemingly similar procedures. For example, when adding fractions, we add numerators while the denominators remain unchanged. Our students might then do something similar when multiplying fractions by multiplying the numerators and keeping the denominators unchanged.

So what is a misconception? A misconception is any idea that is grounded in some degree of understanding but is mathematically flawed. Misconceptions can also be rules or strategies that work in certain situations. For example, we might compare $\frac{2}{8}$ and $\frac{5}{6}$, noting that $\frac{5}{6}$ is greater because we can think about the number of pieces missing from the whole. Yet that strategy doesn't work when two fractions are missing the same number of pieces $\left(\text{e.g.,} \frac{3}{4} \text{ and } \frac{9}{10}\right)$.

Misconceptions are developed through passive, independent observation and incomplete understanding. They can be taught by sharing chants and tricks or even simple reminders. For example, third-grade teachers might "help" their students remember how to record a fraction by saying that the larger number is always on the bottom. This "tip" then causes considerable challenges as students begin to work with fractions greater than 1.

If misconceptions go unnoticed, we are essentially reinforcing them. As teachers, we have to constantly be on the lookout for misconceptions. We have to probe thinking to be sure it is legitimate and complete. We have to be cautious of overreliance on correct answers. As we know, students can arrive at correct answers for the wrong reasons. Yet correct answers can also be found with degrees of correct thinking.

Incomplete thinking can be just as hazardous as a misconception. We might think of incomplete thinking as a *mining hazard.* We assume that everything is fine because student answers and representations seem to indicate understanding. But in fact, our students have found a correct answer for the wrong reasons.

Consider a student who always works with region models of fractions. He consistently finds the shaded part and counts the total pieces. He may begin to associate the idea of a fraction as shaded over total. But in fact, the unshaded portion is also a fraction. He will give correct answers to shaded fraction problems. But he may not have a deeper understanding of fractions. Because of this, he may struggle tremendously

MINING HAZARD

Look for this icon throughout the book. It highlights where students—and sometimes teachers—go awry.

with other models, including number lines, computation with fractions, problem solving with fractions, and eventually ideas about ratio and proportion.

In other situations, a student might rely on repeated addition to find products of multiplication expressions. He seems to understand the meaning of multiplication. Yet when he begins to multiply multi-digit numbers, he is unable to separate from repeated addition in order to apply more efficient methods such as the area model, partial products, or even an algorithm. Also consider working with fractions on number lines. Students who consistently work with endpoints of 0 and 1 may develop incomplete understanding of fractions and have considerable difficulty as fractions greater than 1 are introduced.

Facilitating Discourse

Discourse about our students' ideas and strategies is essential for maximizing the instructional potential of these tasks. These tasks can be applied seamlessly to the five practices Smith & Stein (2011) describe for orchestrating productive discussion:

1. Anticipating student responses
2. Monitoring student work, engagement, and reasoning
3. Selecting student work for discussion
4. Sequencing student responses during discussion
5. Connecting responses and mathematical ideas

Using High-Quality Tasks for Assessment

Quality assessments yield quality information about our students' understanding. These high-quality instructional tasks can easily be used for assessment purposes. We have to keep the purpose of assessment in mind as we use the task. Essentially, we have to know if we will use the task formatively or summatively. In other words, will we use it for instruction (formatively) or for evaluation instruction (summatively)?

What Counts as Evidence?

Regardless of the assessment purpose, we must determine what will count as evidence of understanding before our students work with the task. We might ask if our students will have to compute. Will they be able to justify with a model or drawing? What might their drawings look like? What strategies might they use? Will they have to write sentences to convey understanding? All of these questions represent the thoughts we must consider when determining what counts as evidence of understanding.

This process is similar to anticipating what students will do with a task during instruction. Essentially, we want to determine what will constitute evidence of student understanding. We should make note of specific answers as well as the various strategies our students might use. It is important that we do not confine student responses to what we determine to be evidence.

Understanding Student Thinking and Inference

We can be tempted to infer what students mean when we review student performance on assessments or assessment tasks. This can be *hazardous*. High-quality tasks don't provide opportunities for random answers to be correct. But, as noted,

correct answers are not always the result of correct or complete mathematics understanding. A good rule of thumb is to consider the question, "What would I ask [the student] if she were here right now?" In other words, if we need to ask the student something about her work, her response, or her calculation, it is probably not complete. This doesn't necessarily have to affect a student's score or grade. Instead, it can be an indication of teaching or reinforcing what we need to do with the student.

Determining Student Performance

We might think of our students' performance results in two distinct ways. In one instance, our students demonstrate understanding of the skills and concepts. These students are ready for more opportunities to reinforce their understanding or to advance it to more complex situations. Extending and enriching are other ways to think about advancing. In the other instance, our students demonstrate the need for reteaching.

Reinforce and/or Advance		Reteach	
Student demonstrates full understanding of the concept. The solution is correct. Reasoning is provided through pictures, words, or numbers/equations. Justification is complete. Minor errors may be present but do not affect the response.	Student demonstrates understanding of the concept. The solution may be incorrect but this can be attributed to a computational error rather than flawed logic. Reasoning is provided but may not be complete.	Student demonstrates flawed logic or misconception. The solution may be correct, but it is coincidental.	Student demonstrates no understanding of the concept. The solution is incorrect. There is no justification or reasoning. Numbers or terms are disconnected from the prompt or a restatement of the prompt.

A General Rubric

We know that assessment is much more than counting the number of correct answers on a page. We know that there are layers to every answer. Rubrics are useful because they delineate the layers of understanding and performance. We can design specific rubrics for every task we use. Doing so can be quite daunting. Instead, we may choose to make use of a general rubric that can be applied to most if not all tasks. *We can also connect these rubrics with class observation sheets that capture evidence of student understanding during performance-based or hands-on tasks.*

Using More Than One Task

As we know, it is important to triangulate data points to get a clear picture of where a student is mathematically. This is also true when using high-quality mathematics tasks. These tasks will give better insight into what students know, their partial understanding, and the misconceptions they have. Even so, we should be careful to avoid relying on one task as evidence of student understanding.

Student Performance Recording Sheet Use this with formative assessment tasks, classroom activities, and observations.					
Student Names	Date	Date	Date	Date	Date

Student demonstrates full understanding of the concept. The solution is correct. Reasoning is provided through pictures, words, or numbers/equations. Justification is complete. Minor errors may be present but do not impact the response.

Student demonstrates understanding of the concept. The solution may be incorrect but this can be attributed to a computational error rather than flawed logic. Reasoning is provided but may not be complete.

Reinforce and Advance

Student demonstrates flawed logic or misconception. The solution may be correct but it is coincidental.

Student demonstrates no understanding of the concept. The solution is incorrect. There is no justification or reasoning. Numbers or terms are disconnected from the prompt or a restatement of the prompt.

Reteach

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Other Assessment Tips: Erasing

Erasing is a common practice. We make a mistake and we erase. It may be unavoidable. Yet, we may want to rethink why we erase and if it is always a good idea. When our students erase, they remove a strategy, diagram, or calculation that can provide good insight into their thinking. Knowing what students erase can be just as important as what they didn't erase. Granted, limited space and other confines may affect when our students have to erase. That being said, we may reconsider encouraging students to erase. Instead, we can include their errors as part of the written record.

Other Assessment Tips: Hands-On Tasks

This book provides a collection of paper-and-pencil tasks for instruction or assessment. This is a limitation of the format and medium rather than a message about what we should value in a classroom or what makes a high-quality mathematics task. It is critical that our students have hands-on, concrete experiences with tools, manipulatives, and other resources to learn mathematics. A hands-on task might be an opportunity when students are asked to compare two different fractions using two different tools such as fraction tiles and fraction circles. It's also important to keep in mind that the quality of hands-on tasks can vary wildly as well. The high-quality task identification tool presented earlier in this chapter can be used with hands-on experiences as well.

MINING TIPS

Other tips about the mathematics content, misconceptions, or implementing the tasks are sprinkled throughout the book. This icon signals those ideas.

Hands-on tasks can also be applied to assessment situations. We can use observation rubrics, take notes of student statements and performance, and take pictures of what they do during the activity to document student understanding.

MODIFYING THE TASK

Look for this icon throughout the book. It calls attention to ideas for modifying the associated task.

Modifying Tasks

Miners modify or customize their tools as the mining conditions change. *Each of the provided tasks can be modified*. We might do this so that they better align with our grade-level standards or for a different perspective on student thinking. Each task is provided electronically, and ideas for modification are sprinkled throughout.

REFLECTING ON CHAPTER 1

- How do I typically find and select the mathematics tasks I use in my classroom?
- What are the characteristics of quality mathematics tasks?
- How do I think about the misconceptions my students have? Do I think about them before, during, and after a lesson?
- What misconceptions do I frequently encounter?
- How do I structure my mathematics tasks? Are they teacher centered or student centered? Are there opportunities for collaboration and discussion on a daily basis?
- What are my mining hazards? What do the skills, concepts, or topics that I teach that students seem to understand but do not fully understand?
- What experiences have I had modifying tasks when the topic is related but not completely aligned to what I am teaching?

NOTES

ADDITION AND SUBTRACTION WITHIN 1,000

THIS CHAPTER HIGHLIGHTS HIGH-QUALITY TASKS FOR THE FOLLOWING:

- Big Idea 1: Adding Within 1,000

  Multi-digit addition can be represented with different models, including place value models and number lines. Work with these models builds understanding and lays the foundation for flexible strategies for addition.

- Big Idea 2: Reasoning About Addition Within 1,000

  There is a relationship between addends and sums. Sums change as addends are changed. We can manipulate addends to make addition more friendly. Although addition strategies always work, the efficiency of the strategy relates to the numbers in the situation and the individual's own number sense.

- Big Idea 3: Subtraction Within 1,000

  Multi-digit subtraction can also be represented with place value models and number lines. Subtraction can be thought of as taking away, breaking apart, or comparing two values. We can count back (subtract) or count up (add) to find differences.

- Big Idea 4: Reasoning About Subtraction Within 1,000

  Reasoning about subtraction situations can help us determine accurate differences. To do this, we need to understand the relationship between the minuends, subtrahends, and differences. As with addition, we can manipulate numbers to subtract more efficiently.

- Big Idea 5: Problem Solving With Addition and Subtraction

  Problems can be thought of as any situation in which the solution path isn't apparent. We use addition and subtraction to solve problems. Story problems are one small subset of the types of problems we encounter in mathematics or everyday life. Problem solving requires making sense of the problem, knowledge of strategies, reasoning, and justification.

BIG IDEA 1
Adding Within 1,000

TASK 1A

> **Use a number line to show how you add 358 + 453 and 371 + 361.**

About the Task

We can represent addition with larger numbers on number lines. However, the size of these numbers limits the model to an open or empty number line. *These number lines do not have tick marks for each number.* In some cases, these number lines do not have defined endpoints either. In this task, students add different three-digit numbers on an open number line. The openness allows them to apply flexible strategies to the computation.

Anticipating Student Responses

Students are likely to decompose one or both numbers by their place value. For 358 + 453, these students may begin with 358 and make a jump of 400 (to 758), a jump of 50 (to 808), and then of 3 (to 811). *Some students may decompose an addend and then make repeated jumps of the place value. In other words, a jump of 400 would be represented by four jumps of 100.* Other students may jump by place value but begin with the ones place. They would first make a jump of 3, then 50, and then 400. Some students may make endpoints of 0 and 1,000 on their number line. Other students may assign one of the addends to the left endpoint and then jump/count on from there. Some of our students may find the sum of the numbers with an algorithm or similar procedure and then represent the addends and sum on the number line.

MINING TIP

It is important to connect ticked number lines with open number lines to support our students' transition to open number lines. We can adjust the intervals of tick marks to support the transition. For example, we can change the intervals from 1, 2, 3 to 10, 20, 30 or 100, 200, 300.

MINING HAZARD

Students who make a jump of 400 as four jumps of 100 are mathematically accurate. However, this strategy is less efficient than making one jump of a larger amount.

PAUSE AND REFLECT

- **How does this task compare to tasks I've used?**
- **What might my students do in this task?**

Visit this book's companion website at **resources.corwin.com/minethegap/3-5** for complete, downloadable versions of all tasks.

WHAT THEY DID

Student 1

Student 1 shows that he doesn't understand the meaning of the equations. He adds up from one addend to the other. This would be a viable strategy for finding the difference between two numbers on the number line. We can be encouraged that he makes use of friendly numbers in the first prompt.

Student 2

Student 2 decomposes the addends into unique chunks. For the first prompt, he jumps by 50, then 40, and then two jumps of 5. The sequence is equivalent to 100 but slightly more complicated. We can also note that he mixes in two jumps of 5 before then adding a large jump of 300. In the second prompt, he jumps by 200 before breaking apart the remaining 100 to smaller jumps. His mathematics is accurate but inefficient.

USING EVIDENCE

What would we want to ask these students? What might we do next?

Student 1

Our first action with Student 1 is to ask him to describe the meaning of the expressions. It is possible that he misread the problem, thinking subtraction instead of addition. Assuming that he did read it correctly, we know that we have work to do to develop understanding of the expressions and the operation. *It would be wise to put the expression into context and work with models of the quantities with base ten blocks or similar models.* We can work to count up by place values, making use of expanded form. We can compare the sum of the base ten blocks with the representation and location on the number line.

Student 2

It is likely that Student 2 is quite comfortable manipulating numbers. His complicated jumping may be a "look what I can do" statement. It's also possible that he has a notion of benchmarks and is trying to navigate them through the computation. For example, on the first number line, he jumps to 408, which is close to the 400 benchmark. His next jump of 40 lands him at 448, which is also close to the benchmark of 450. *Student 2 serves as a reminder that our students can exaggerate ideas of our mathematics instruction.* We should discuss with him how to add parts of the addend with fewer jumps. We may also want to shift focus to decomposition of numbers to friendly chunks. For example, 453 might be thought of as 400 + 50 + 3 or 450 + 3.

Student Work 1

Add 358 + 453. Use the number line to show how you added.

+2   +40   +53
358   360   400   (453)

Add 371 + 361. Use the number line to show how you added.

+10
361   (371)

Student Work 2

Add 358 + 453. Use the number line to show how you added.

+50   +40   +5   +5   +300   +50   +3
358   408   448   (453)   458   758   808   (811)

Add 371 + 361. Use the number line to show how you added.

+200   +30   +60   +10   +60   +1
371   571   601   661   671   731   (732)

WHAT THEY DID

Student 3

Student 3 is accurate in his jumps. Unlike Student 2, he decomposes the addends into reasonable chunks. His first prompt shows reliance on skip-counting by hundreds before adding a multiple of 10. His work with the second prompt shows a jump of a multiple of 100 (300) and then a multiple of 10 (60). The jump of 60 results in a miscalculation.

Student 4

Student 4 decomposes by place value. He adds the partials accurately. The first prompt is likely less complicated because 50 is added to 758. However, he is equally successful when he wraps around the century by adding 60 to 371. *We can see that his jumps are not proportionate, especially with the jumps of 3 and 1 in the first and second prompts, respectively.*

USING EVIDENCE

What would we want to ask these students? What might we do next?

Student 3

Student 3's work in the first prompt shows comfort, possibly preference, for counting on by hundreds. His next jump of 50 is likely connected to his recognition of 758 and understanding of adding fifties (758 and 50). *This is evidence of comfort with making hundreds. We should connect this idea with his response in the second prompt, where he tries to "wrap around" the century (671 + 60).* Instead, we might help him think about how many are needed to get to the next hundred (30) and then add the rest of the tens from that point.

Student 4

Student 4 shows proficiency with adding on a number line. He also shows proficiency with breaking apart addends to add more efficiently. We can begin to challenge him with more advanced ways of decomposing numbers as he shows readiness. For example, he may be ready to think of 361 + 371 as 350 + 350 + 11 + 21, which becomes 700 + 32 by redistributing the amount within each addend. In the first prompt, he might think of 358 + 458 as 350 + 350 + 100 + 16 or 350 + 450 + 16.

Student Work 3
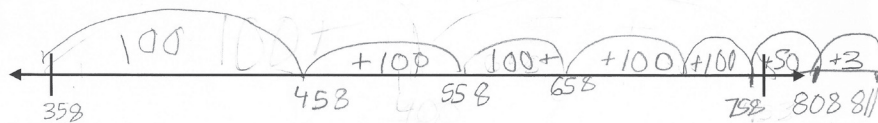
Add 358 + 453. Use the number line to show how you added.

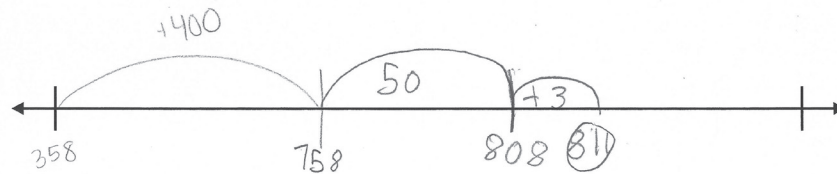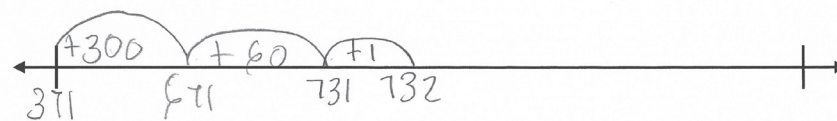

Add 371 + 361. Use the number line to show how you added.
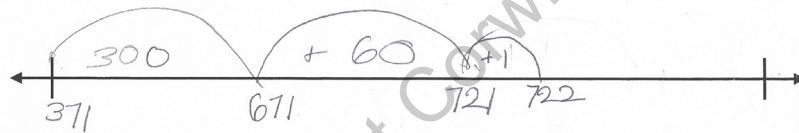
Student Work 4

Add 358 + 453. Use the number line to show how you added.

Add 371 + 361. Use the number line to show how you added.

- **What will count as evidence of understanding?**
- **What misconceptions might you find?**
- **What will you do or how will you respond?**

Visit this book's companion website at **resources.corwin.com/minethegap/3-5** for complete, downloadable versions of all tasks.

TASK 1B: Dennis got out some base ten blocks (235). Jackie got out some base ten blocks (137). How many blocks did Dennis and Jackie get out all together? Use pictures, numbers, or words to explain your answer.

MODIFYING THE TASK

We can extend the task by asking students to create an addition problem and then represent it with base ten blocks. Students can draw/represent base ten blocks with dots (ones), lines (tens), and squares (hundreds).

Addition within 1,000 should build from conceptual understanding of numbers and the operation as it does with smaller addends. We can use similar tools to develop this understanding. *This problem prompts students to add two three-digit numbers represented by base ten blocks.* The blocks are not arranged in order of place value. Students will need to show that they have made sense of each number. They will also need to show how they found their sum. Some students may simply count all of the blocks for each place value to find the sum. Doing so shows that they understand the meaning of addition, but it also shows that they rely on a lower-level strategy (counting on) and physical models or drawings to combine larger numbers. It will be important to explicitly connect the representations with computation on number lines and equations.

TASK 1C: Kelly added 348 + 256 by breaking both numbers apart. She created 300 + 40 + 8 and 200 + 50 + 6. She said she could then just add the hundreds, tens, and ones to get the sum. Do you agree? Will this always work? Create a new equation to show if it will or won't work.

Flexibly decomposing numbers enhances our ability to compute efficiently and mentally. In this task, students are asked to consider if we can decompose two addends by their place values and then add by their place values. Student understanding of decomposition, in this case expanded form, may be their greatest challenge. Students who understand this will note that you are still adding the same numbers, so it does and always will work. Others will state that there is a difference between the number (348) and the expanded form of it (300 + 40 + 8). The task notes that the numbers are then added by hundreds, then tens, and lastly ones. Students may believe that you can only add by starting with the ones place. Yet, this is only necessary when applying a traditional algorithm. For these students, we can add using the expanded form of two addends starting with ones and then add again starting with hundreds, noting that the sum remains the same regardless of which place value we begin with.

TASK 1D: Use a hundred chart (701–800) to add 732 + 59. Use a hundred chart (501–600) to add 514 + 77.

Hundred charts are quite useful for adding within 100. We can modify them to model addition within different centuries. In this task, students use a 700 chart and 500 chart to add numbers within the respective century. Some students may count on from one addend by ones. Many students are likely to add by tens and then ones or by ones and then tens. We should look for students who make a jump of a multiple of ten. Doing so shows a more refined approach to computation. We can connect the computation represented on hundred charts with it represented on number lines. This helps students transfer their understanding to new models. We can also connect it to equations to lay the groundwork for developing understanding of symbolic representations and eventually an algorithm.