Operating With Matrices

1. Which of the following is the simplification of the given matrix expression?

$$\begin{bmatrix} 3 & 1 & 2 \\ 4 & 2 & 0 \end{bmatrix} \cdot \begin{bmatrix} 5 & 3 \\ 1 & 2 \\ 7 & 6 \end{bmatrix}$$

a. $\begin{bmatrix} 15 & 1 & 14 \\ 12 & 4 & 0 \end{bmatrix}$

b. $\begin{bmatrix} 15 & 12 \\ 1 & 4 \\ 14 & 0 \end{bmatrix}$

c. $\begin{bmatrix} 30 & 23 \\ 22 & 14 \end{bmatrix}$

d. Can't be done

Explain your choice:

2. Which of the following is the simplification of the given matrix expression?

$$\begin{bmatrix} 0 & 5 & -2 \\ 1 & -3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 & 6 \\ 6 & -1 & -4 \end{bmatrix}$$

a. $\begin{bmatrix} 0 & 15 & -12 \\ 6 & 3 & -16 \end{bmatrix}$

b. $\begin{bmatrix} 3 \\ -7 \end{bmatrix}$

c. $\begin{bmatrix} 2 & 15 & -12 \\ 6 & 3 & -16 \end{bmatrix}$

d. Can't be done

Explain your choice:

Teacher Notes: Operating With Matrices

Questions to Consider About the Key Mathematical Concepts

Can students correctly multiply matrices and recognize when matrix multiplication cannot be performed because of the sizes of the matrices? To what extent do they

- make sense of matrix dimensions, entries, and operations with matrices?
- reason whether two matrices can be multiplied?
- describe the process of multiplying two matrices and why specific dimensions are needed?

Common Core Connection (CCSS.Math.Content.HSN-VM.C.8)

Grade: High School

Domain: Number and Quantity

Cluster:

Perform operations on matrices and use matrices in applications.

C8. Add, subtract, and multiply matrices of appropriate dimensions.

Uncovering Student Understanding About the Key Concepts

Using the Operating With Matrices Probe can provide the following information about how the students are thinking about operations on matrices.

Do they

- recognize the correct dimensions needed for matrix multiplication (the number of columns in the first matrix needs to be the same as the number of rows in the second matrix)?

OR

- correctly multiply the rows in the first matrix with the columns in the second matrix?

Do they

- think matrices of any dimension can be multiplied?

OR

- not understand matrix multiplication to be a specific process?

Do they

- understand the result of multiplying a 2 × 3 matrix with a 3 × 2 matrix to be a 2 × 2 matrix?

OR

Do they

- think the resulting matrix will be the dimensions of one of the original matrices?

Exploring Excerpts From Educational Resources and Related Research

Areas of consideration:

Multiplying two matrices can be challenging for many students. They must be able to indentify which matrices can be multiplied and they often have trouble with the concept that multiplication with matrices is not commutative. (Muschla, Muschla, & Muschla-Berry, 2011, p. 132)

Too often, students view matrices as nothing more than abstract rows and columns with which they demonstrate their arithmetic skills. They need to go beyond these manipulations and acquire the knowledge needed to connect matrices with the real world. (Worrall & Quinn, 2001, p. 46)

Geometric introduction to matrices [can] strengthen students' conceptual understanding of matrix algebra. Such concepts as matrix multiplication seemed easier for students when they were first approached in a visual manner. (Edwards, 2003, p. 48)

Surveying the Prompts and Selected Responses in the Probe

The Probe consists of two selected response items. The prompts and selected responses are designed to elicit understandings and common difficulties as described below:

If a student chooses	*It is likely that the student*
1. c and 2. d (correct responses)	• understands how to multiply matrices of correct dimensions. The column dimension of the first matrix needs to match the row dimension of the second matrix (the number of columns in the first equals the number of rows in the second). The resulting matrix will be the row dimension of the first and the column dimension of the second. In Example 1, the first matrix is a 2 × 3 and the

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If a student chooses	*It is likely that the student*
	second is a 3 × 2. As these are dimensions that can be multiplied, the resulting matrix will be a 2 × 2. Each entry in the new matrix is obtained by multiplying the row entries in the first matrix with the column entries in the second matrix, then adding the numbers. For instance, to get the first entry in the resulting matrix, you would do the following computations: (3)(5) + (1)(1) + (2)(7), which equals 15 + 1 + 14 = 30. [See Sample Student Response 1] *Look for indication of the student's understanding in the written explanations of how the student got the answer.*
1. a or b	• does not fully understand the concept and process of matrix multiplication. The student is multiplying numbers but not in the correct way. As the process is a bit complicated, students may or may not be on their way to complete understanding. Asking the student clarification questions will help to understand where they are in their understanding. [See Sample Student Response 2]
1. d 2. a, b, or c	• does not understand that in order for matrices to be multiplied, the number of column entries in the first matrix needs to be the same as the number of row entries in the second matrix. [See Sample Student Response 3]

Teaching Implications and Considerations

Ideas for eliciting more information from students about their understanding and difficulties:

- What are the dimensions of each matrix? Is the number of rows or columns represented first in a matrix dimension? What might help you remember that the first number in a matrix dimension represents the number of rows in the matrix?
- What needs to match in order to multiply matrices? Why?
- If two matrices have appropriate dimensions to be multiplied, how can you tell what the dimensions of the resulting matrix will be? (The number of rows of the first and the number of columns of the second)
- Should you circle the rows or the columns in the first matrix? (rows) Why?
- Should you circle the rows or columns in the second matrix? (columns) Why?
- If you have matrix *A* and matrix *B*, is multiplying *A* times *B* the same as multiplying *B* times *A?* Why or why not?
- If multiplying matrix *A* times matrix *B* is possible, does that automatically mean that multiplying matrix *B* times matrix *A* is also possible?

Ideas for planning instruction in response to what you learned from the results of administering the Probe:

- When talking about entries, use words and notation that will help students know what is being discussed. For example in the second matrix in Question 1, when talking about the entry where the 7 is, say "row 3, column 1." This allows them to become familiar with rows, columns, and the placement of each entry. This also helps when finding the entries in the multiplied matrix. As the final matrix is a 2 × 2, if you are looking for the entry in row 1, column 2, you will be multiplying numbers in row 1 of first matrix with numbers in column 2 of second matrix.
- As the row entries of the first matrix need to be multiplied by the column entries of the second matrix, have students circle the rows in the first matrix and the columns in the second matrix. Colored pencils work well for students who like colors!
- Include letters (variables) when working with computations on matrices. This allows students to think abstractly and allows for discussions on conceptual understanding of the processes being used.
- To build conceptual understanding of operation with matrices, use matrices to explore real-world contexts. The use of technology "allow the students to extend their investigations into more interesting and complex situations without having to continuously complete tedious calculations" (Worrall & Quinn, 2001, p. 49).

Sample Student Responses to Operating With Matrices

Responses That Suggest Understanding

Sample Student Response 1

Probe Item 1: Student chose c. $\begin{bmatrix} 30 & 23 \\ 22 & 14 \end{bmatrix}$. Match up the numbers. The 30 comes from multiplying 3 × 5 and 1 × 1 and 7 × 2 and then adding those 3 numbers. Just move over to get the next number. Multiply the 3, 1, and the 2 by the 3, 2, and 6 in the next matrix. Finally move to the 4, 2, and 0 and repeat in the same way.

Responses That Suggest Difficulty

Sample Student Response 2

Probe Item 1: Student chose a. $\begin{bmatrix} 0 & 15 & -12 \\ 6 & 3 & -16 \end{bmatrix}$. It's easy. Just multiply each number in the same spot to get a new number.

Sample Student Response 3

Probe Item 1: Student chose d. Can't be done. The numbers of rows and columns don't match.