Chapter 5

To Teach or Not to Teach Mathematical Algorithms

During the twentieth century, educators waged a number of battles over how to teach mathematics. During the middle of the century, educators argued over whether it was more important for children to *understand* mathematics or to be able to *do* mathematics—whether conceptual or procedural knowledge was paramount. Eventually educators decided that both were important and that it was also imperative for children to understand the relationship *between* their conceptual and procedural knowledge.

By the last decades of the twentieth century, another battle had commenced. The new battle arose because mathematics educators discovered a philosophy called "constructivism" and a new type of knowledge called "meanings." With the publication of *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989), educators increasingly demanded that instruction focus on helping children construct their own personal meanings about mathematics—in contrast to helping them learn the accumulated knowledge created by mathematicians over the centuries.

With the gradual introduction of mathematical meanings into the school curriculum, the battles intensified about the types of knowledge children should learn in school. Should children *understand* mathematics, be able to *do* the mathematics needed to function productively in society, or construct their own personal mathematical *meanings*? The battle over *procedural* knowledge (skills) versus *constructivist* knowledge (meanings) was most visibly fought in California in the last few years of the twentieth century. In that conflict *whole math* and *whole language* were pitted against a *skills* approach to mathematics and a *phonics* approach to reading. Traces of that battle can be found at Web sites such as www.mathematicallycorrect.com. The battle between conceptual knowledge (understandings) and constructivist knowledge (meanings) was fought for over a decade under the motto "back to basics." Traces of that conflict can be found at Web sites such as www.wgquirk.com.

These battles continue to be fought today. There are those in the constructivist movement who condemn any direct teaching of skills as harmful to children. Similarly, there are those in the skills movement and the understanding movement who condemn constructivist teaching as destructive to society.

In presenting an oral story that teaches the multidigit addition algorithm, this book steps into the middle of the battle over whether understandings, skills, or meanings should be the primary type of mathematical knowledge taught in schools. In suggesting that the algorithm can be meaningfully taught, it directly confronts a pedagogical position that is currently fashionable among a number of mathematics educators who have written about the harmful effects of teaching children arithmetic algorithms and who have suggested that we stop teaching algorithms in our schools and instead allow children to invent their own methods of solving problems (Kamii & Dominick, 1998; Burns, 1994; Madell, 1985). While the reasons given for the harmful effects of algorithms are problematic in and of themselves, it is equally distressing that the challenge to those of us who do believe that algorithms can be meaningfully taught to children has not been met and debated openly in the professional literature.

This book confronts the claim that the teaching of algorithms in schools is harmful to children. It does so by providing a counter example to the claim that algorithms cannot be taught in a meaningful and enjoyable way by presenting and discussing "The Wizard's Tale." "The Wizard's Tale" demonstrates one way—and by no means the only way—of meaningfully teaching addition.

This chapter also confronts the assertion that algorithms should not be taught in schools by claiming that the teaching of algorithms is a complex task that requires the acquisition of understandings, skills, and meanings and, in addition, that it is important for children to meaningfully construct for themselves the relationship between their skills, understandings, and meanings. The teaching of algorithms is not a simple task that requires drawing on only one dimension of our multidimensional mathematical knowledge base. It is a delicate balancing act in which the teaching of understandings, skills, meanings, and values must be carefully coordinated, sequenced, and synchronized. In so doing, this book implicitly takes the position that we do not need to choose between teachers teaching algorithms and children inventing their own algorithms, but that these two activities can complement and enrich each other.

Educators' assertions about the harmful effects of teaching algorithms might be justified if they were aimed at direct teaching that promotes rote mechanistic learning of decontextualized rules (for example, in division "estimate, multiply, subtract, . . ."). The attack, however, seems to be aimed at *all* methods of teaching algorithms and is thus deemed inappropriate. Also inappropriate is the solution to the problem of how to help children learn algorithms, which seems to be to discourage teachers from teaching algorithms.

To provide children with only rote, mechanistic, decontextualized direct instruction, and no time to construct mathematical meanings on their own, is problematic, given what we know about how children construct mathematical meanings over an extended period of time based on multiple experiences. But to withhold from children algorithms that have been culturally constructed and refined through centuries of mathematical endeavors, and to prohibit teachers from providing children with meaningful direct instruction, is also problematic, for it withholds from children their cultural heritage.

We need to provide children with an appropriate mix of both meaningful direct instruction and student discovery. In the case of multidigit addition (which is featured in "The Wizard's Tale"), as well as other operations, instruction should contain at least three types of experiences that children repeatedly encounter.

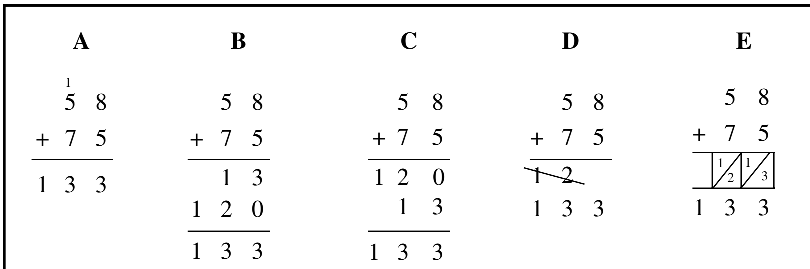
One type of experience involves introducing children to the meaning of multidigit addition by extending beginning addition concepts to multidigit numbers and connecting them to place value concepts. Children should work with a variety of real-world and fantasy situations in which they are encouraged to invent their own algorithms, share them with classmates, and compare such things as the accuracy, ease of comprehension, and efficiency of their inventions. They should work a wide range of problems using discrete materials (such as apples) and continuous quantities (such as distance). They should work with real-world materials (such as pennies), physical representations (such as unifix cubes), pictorial representations, and numerical symbols. Place value concepts should be introduced using a variety of manipulatives: materials that children themselves assemble into groups of ones, tens, and hundreds (such as multilink cubes); structured materials already assembled to graphically represent ones, tens, and hundreds (such as base ten blocks); and nonproportional materials (such as money). Children should use these materials to invent ways of acting out problems, such as packaging factory inventory in groups of ones, tens, and hundreds or buying and selling groups of items from a classroom store. Problems should be worked on and off of place value mats and should be drawn from a variety of sources, including real-world situations, children's literature, teacher-generated problems, oral stories created by teachers, and written and oral stories invented by children. Children should solve problems, represent solution strategies, and communicate their ideas by doing such things as role-playing with manipulatives, orally describing their inventions, drawing pictures of their endeavors (sometimes using ink stamps and ink pads), and writing descriptions of their procedures.

A second type of experience involves teacher-directed presentation of one or more standard multidigit addition algorithms. These might arise in the context of real-world or fantasy situations. One way to do this is by telling children oral fantasy stories in which the traditional algorithm is introduced in a meaningful context in such a way that the mathematical structure and meaning of the algorithm is clearly presented. "The Wizard's Tale" is one example of such a story. It lasts several days and involves careful building of the routines that children need to learn. In addition, children participate in the story by playing roles in which they are recorders of actions with paper and pencil (the gorilla), physical manipulators of place value manipulatives (the bulldozer), and verbal narrators that describe the actions and recordings taking place (the parrot).

A third type of experience involves helping children to meaningfully generalize their understandings and skills and to see the underlying structure of mathematics by comparing addition solution strategies when using such diverse things as mental calculations, alternative algorithms (such as those presented in Exhibit 5.1), 0 to 99 number charts, calculators, and the standard algorithm. While engaging in this type of activity, children should be encouraged to explain, compare, and relate different solution procedures (including their invented algorithms, alternative algorithms, and the standard algorithm). Being able to reconcile the differences between these solution strategies and comprehend and explain how all of them accomplish the same task in a roughly similar manner is an important part of understanding multidigit addition.

These three types of experiences should be encountered repeatedly by children, nudging them to meaningfully construct increasingly greater understanding of addition and addition skills with each experience. Taken together, the three types of experiences (and possibly others) would infuse increased meaning into the addition process. They could do so by helping children construct a rich understanding of addition through many different types of

Exhibit 5.1



Addition Algorithms

A is the "currently traditional" right to left algorithm.
B is a variation on A that uses partial sums.
C is a version of B that is worked from left to right.
D is a left to right version of C in which partial sums are updated during addition.
E formats recording so that "trades" between columns are usually unnecessary.

experiences, encouraging children to invent their own algorithms in an environment in which they have to explain the underlying mathematics of their creations to their peers and teachers, helping them learn the standard algorithm, helping them acquire skills in performing addition, and cumulatively connecting new learnings to previous ones in meaningful ways. They would encourage children to think deeply about mathematics as they construct their own meanings and compare different algorithms in order to help them understand the underlying structure of mathematics and build important mathematical skills. And many different instructional techniques—including mathematical oral storytelling—would be embraced to help children with a variety of learning profiles, rather than relying only on one method that must work for all children. In so doing, meaningful teacher-directed instruction and instruction that encourages children to create their own invented algorithms would work together to help children construct a rich, multidimensional understanding of the standard algorithm or whichever algorithm is deemed appropriate for children to learn.