2

GENERALIZATIONS OF REGRESSION 1

Testing and Interpreting Interactions

The model of the previous chapter is ***additive*** in the sense that it envisions the effects of variables as *independently* and cumulatively adding to (or subtracting from) the outcome. Each variable operates independently because the effect of each *X* applies at all levels of other (*X*) variables.

The first limitation we focus on in this chapter is this issue of additivity. Imagine how often you hear someone say in response to generalizations: "Not necessarily. It depends." That hypothesis expresses the prevalence of *conditional* effects in our thinking. In other words, whether X_1 affects *Y* depends on the level of another variable X_2. These are called *interactions*, in regression model terms. ***Interactions express the possibility that the effect of a particular X changes depending on the level of some other X in the equation***.

2.1 INTERACTIONS IN MULTIPLE REGRESSION

Interaction Defined. An interaction exists when the ***effect*** of a chosen focal variable *X changes* depending on the level (or category) of a second variable (here called *Z* to distinguish roles). Interactions are expressed in regression equations by forming multiplicative terms between *X* and *Z*, such as *X* • *Z*. As shown below, this term expresses the possibility that the effect of each variable changes depending on the level of the other.

Here is an example of an equation with an interaction between *X* and *Z*:

$$Y_i = a + b_1 X_i + b_2 Z_i + b_3 (X_i \cdot Z_i) + e_i$$

Going forward, we will not show the "*i*" subscript for individual observations in these equations, unless the method we discuss demands a modification. In this definition, note that the effect of *X* now appears twice in a model—on its own and as part of a variable in which *X* is multiplied by *Z*. Now the effect of *X* is captured collectively by both variables—*and only by both variables*—because this effect changes depending on the level of *Z*. The *b* for the multiplicative variable here literally asks, Will the effect of X change across levels of Z (*and vice-versa if we choose to make Z the focal variable*)?

2.1.0.1 Importance of Interactions

We begin with interactions in regression because of their prevalence in our theorizing, the flexibility they introduce into regression, and the importance of avoiding the over-generalization of findings. Note these points about the ubiquity of interactions in theory, in qualitative sociology, and across a range of techniques and disciplines:

- Interactions are predicted whenever theories express historical, spatial, or cultural boundaries or social conditions that make the theory salient.
- Historical explanations commonly use interactional theorizing: Y happened only because both X and Z were present.
- Theories and qualitative perspectives, such as intersectionality, often pose the necessity of multiple conditions for an outcome to occur. If the effect of sex depends on race and class and cannot be discussed on its own, this *is* an interaction.
- Interactions take into account sources of individual identifiers as *modifiers* of the relevance of variables we study and thus help avoid overgeneralization.
- Many techniques rely on interactions for the testing of basic and important hypotheses about generalizing across societies, groups within societies, or over time.
- Interactions are the ***only*** way to test the uniqueness of specific effects in one group–for example, women, immigrants, visible minorities—relative to others. One-group studies cannot really do this.
- The study of separate subgroups (men vs. women, Black vs. White, etc.) *directly implies that an interaction exists for some variable*. You should always test an interaction ***before*** implying different effects in subgroups. In other words, you should not simply set up analyses in separate subgroups presumptively—it leads to the impression of uniqueness without testing it.

On the last point, we take the position throughout, following prevailing wisdom, that testing interactions in a full sample is preferable to splitting a sample into subgroups in order to compare coefficients across samples—for a list of reasons, see Williams (2015). There are technical issues to consider, such as statistical power to detect differences in the effect of X, but we emphasize an additional issue: Testing interactions in a single equation in a full sample encourages a theoretical focus on the combined effects of specific variables rather than the entire model while also allowing access to all of the information available in an analysis of split sub-samples. This last point may not be widely realized, and thus we focus on *completely* interpreting interactions in this chapter.

2.1.1 Two-Way Interactions Between Continuous and Categorical Variables

We begin with a classic case: The effect of a continuous variable X varies across categories of a categorical variable standing for different groups (here denoted by Z)—for example, race/ethnicity, gender, employment status, nativity, religion, occupation, marital status, etcetera.

Typically, these variables are coded originally as a single categorical variable with arbitrary numerical codes to make distinctions among groups. The ordering of the numbers means nothing. For example, in the simplified example below, race/ethnicity is assumed to be coded 1 = Black, 2 = Hispanic, and 3 = White. There is no ordering implied by these numbers.

To treat this categorical variable in an interaction, you develop a set of dummy variables standing for membership in each of the groups contained in the categorical variable—following the procedures of the previous chapter. Ordinal variables have ordering but not equal distance among categories, and as a result, they usually have to be turned into sets of dummy variables as well.

2.1.1.1 Example: The Long-Term Effects of Education on Personal Income

Suppose we are interested in predicting current personal income, coded in thousands of dollars. This is the dependent variable in this example.

We are interested in the detection of discrimination effects by racial / ethnic categories. One manifestation of discrimination may be the lower average education of minority groups, but another may be that the income *returns* to education for minority groups are lower than for the White majority. Translated, this hypothesis implies an interaction between education and racial group in predicting income, such that the largest impact of education occurs among Whites, and we see a significantly reduced impact among minorities.

To make the example more realistic, we also control for *years in the current job*, since job seniority will also predict current income.

There are three independent variables:

1. **R: race**, Black / Hispanic / White (other groups are excluded from the analysis)
2. **E: education,** in years
3. **S: job seniority,** years in current job

We need to create dummy variables to represent the racial categories. Following the discussion in Chapter 1, the most general rules for creating dummy variables here are:

1. For a categorical variable with *k* categories, you need to form *k* – 1 dummy variables to stand for differences among all *k* groups. One group is left out and is known as the reference group or category.
2. Choose a reference group that makes interpretation easier: This could be the highest or lowest group on the dependent variable or a natural comparison group (e.g., non-employed for employment status, native-born for immigration status) or a middle group standing for normal, equitable, or usual. It is up to you as the analyst.

In this case, we will create two dummy variables for race. Table 2.1 shows the coding of the original race variable and the two dummy variables derived, with B for Black, and H for Hispanic. White is the reference category.

TABLE 2.1 ■ CODING RACE/ ETHNICITY INTO DUMMY VARIABLES

		Dummy Variables	
	Race Coding	B	H
Black	1	1	0
Hispanic	2	0	1
White	3	0	0

In this coding scheme, you will actually "see" variables for Black and Hispanic, but not "White." Each variables is coded = 1 to stand for that group and 0 for all other groups. When considered as a group, each variable expresses the mean difference on *Y* of that group versus the reference group, in this case, Whites.

There are a number of questions that we could ask in this analysis. First we can test the predicted interaction by forming interaction terms between the continuous variable *E* and each of the dummy variables, specifically *E* • *B* and *E* • *H*. We need to include *both* terms in the regression at once to test the ***overall*** interaction between education and race. This is not a small issue: It is important to test the overall interaction *first* before interpreting specific terms in the interaction involving specific groups. You cannot just search for a significant term in the overall interaction and discuss this as a "significant" interaction.

In addition, if the interaction is not significant, implying that education has the same effect on income in all three groups, we would also want to know if there are general group differences in income controlling for level of education—another form of a discrimination effect.

Note something important about interactions: We are focusing on the *impact* of *X* and how it changes, not the average difference in *X* across the groups.

2.1.2 Procedure for Testing an Interaction

1. Begin with a basic additive model for the effects of education, race, and job seniority. This includes **E, B, H,** and **S**

$$\hat{Y} = a + b_1 E + b_2 B + b_3 H + b_4 S \quad (1)$$

2. In a second model, add *both* two-way interaction terms for education and race:

$$\hat{Y} = a + b_1 E + b_2 B + b_3 H + b_4 S + b_5 (E \cdot B) + b_6 (E \cdot H) \quad (2)$$

As pointed out above, to specify the overall interaction completely, you need to multiply the continuous variable by ***each*** of the dummy variables in the equation representing the underlying categorical variable.

To test the current interaction, you need to assess the R^2 increase in Model 2 relative to Model 1 with an ***F*-test**. If the increase in R^2 is significant, retain and interpret model (2). If not, interpret Model 1. This test assesses whether unique explained variance in *Y* is added to the equation by the interactions, because Model 1 is "nested" in Model 2, and so the difference in R^2 can only reflect the impact of the added interaction terms.

We estimate these models in the National Survey of Families and Households. The *F*-test you use here is shown below, with subscripts standing for the model. The "*k*" in this formula refers to the number of independent variables in each model.

$$F = \frac{R^2_{(2)} - R^2_{(1)} / k_2 - k_1}{1 - R^2_{(2)} / N - k_2 - 1} \text{ with } k_2 - k_1 \text{ and } N - k_2 - 1 \text{ df}$$

The R^2 in Model 2 is .2285. The R^2 in Model 1 is .2218. k_2 is the number of independent variables in Model 2 (6). k_1 is the number of independent variables in Model 1 (4). and the sample size is 5472. The *F* value we calculate is

$$F = \frac{(.2285 - .2218)/2}{(1\text{-}.2285)/5472\text{-}6\text{-}1} = 23.73 \text{ with 2 and 5465 df}$$

This is significant beyond the .0001 level; thus the interaction between education and race is significant. The following estimates are found for Model 2.

$$\hat{Y} = -26.576 + 3.142E + 14.15B + 19.67H + .615S - 1.39(E \cdot B) - 1.76(E \cdot H)$$

When there are interactions in the equation, you cannot interpret the so-called "main effects" of any of the variables involved in the interaction in the way you usually would. For example, as we shall see, education does not generally increase income by 3.142 thousand dollars per year of education, as the coefficient suggests. This effect pertains to *only one of the three racial categories—the reference group of Whites.*

To interpret interactions properly, you need to analyze the equation to derive the specific effects of education in each of the three groups. To do this, plug values into the equation standing for each of the three groups and solve for the effect of the continuous variable *E*.

Seniority is a control variable in this equation that is extraneous to the interaction and does not affect it. Thus we can set it to its mean value throughout our calculations so that it does not affect our interpretation. This mean can be found from the descriptive statistics for the regression (here it is 13.98). Note that this will adjust the intercept to a new value that will apply throughout the calculations. The mean value for **S** is substituted in each of the calculations in Table 2.2.

TABLE 2.2 ● DERIVING SUBGROUP SLOPES IN A TWO-WAY INTERACTION

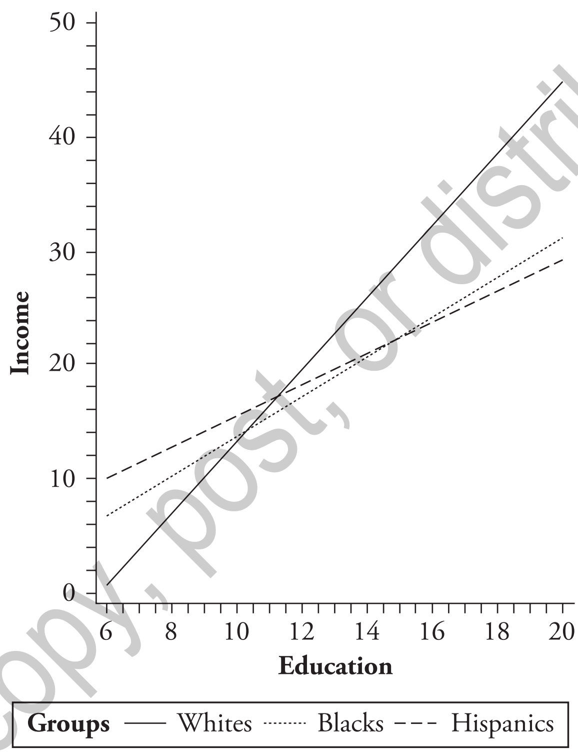
Group	B	H	Deriving Subgroup Equations from $\hat{Y} = a + b_1E + b_2B + b_3H + b_4S + b_5(E \cdot B) + b_6(E \cdot H)$
White	0	0	$\hat{Y} = a + b_1E + b_2 0 + b_3 0 + b_4(13.98) + b_5(E \cdot 0) + b_6(E \cdot 0)$ $\hat{y} = a + b_4(13.98) + b_1E$ Substituting coefficients and simplifying: $\hat{y} = -26.576 + .615(13.98) + 3.142E$ $\hat{y} = -17.97 + 3.142E$
Black	1	0	$\hat{Y} = a + b_1E + b_2 1 + b_3 0 + b_4(13.98) + b_5(E \cdot 1) + b_6(E \cdot 0)$ $\hat{y} = a + b_1E + b_2 + b_4(13.98) + b_5E$ $= (a + b_2 + b_4(13.98)) + (b_1 + b_5)E$ Substituting and simplifying: $\hat{y} = (-26.576 + 14.15 + .615(13.98)) + (3.142 - 1.39)E$ $\hat{y} = -3.828 + 1.752E$
Hispanic	0	1	$\hat{Y} = a + b_1E + b_2 0 + b_3 1 + b_4(13.98) + b_5(E \cdot 0) + b_6(E \cdot 1)$ $\hat{y} = a + b_1E + b_3 + b_4(13.98) + b_6E$ $= (a + b_3 + b_4(13.98)) + (b_1 + b_6)E$ Substituting and simplifying: $\hat{y} = (-26.576 + 19.67 + .615(13.98)) + (3.142 - 1.76)E$ $\hat{y} = 1.696 + 1.386E$

Follow this procedure to substitute values: (a) substitute values for each group in the interaction equation in turn and remove all terms multiplied by zero; (b) collect terms multiplying the same variable; (c) substitute actual coefficients from the results and calculate the slope for ***E*** in each group and the adjusted intercept in each group. Note that every term will have one of two roles: an adjustment to the slope of ***E*** or an adjustment to the intercept in that group.

If you just look at the coefficients for the effects of education, you can see the reduced impact of education among Blacks and Hispanics. Whites receive about \$3,142 per year of education, Blacks receive just \$1,752, and Hispanics receive even less, about \$1,386 on average.

Each equation could be graphed to show the differences in effects, as shown in Figure 2.1.

FIGURE 2.1 ⬢ A TWO-WAY INTERACTION BETWEEN EDUCATION AND RACE



Education has its strongest impact on eventual income among Whites, and both Blacks and Hispanics receive significantly lower returns to education. This can also be shown by the results from the regression using SAS output in Table 2.3.

TABLE 2.3 ⬢ REGRESSION RESULTS FOR THE TWO-WAY INTERACTION

Variable	Label	DF	Parameter Estimate	Standard Error	*t* Value	Pr > |t|
Intercept	Intercept	1	−26.57613	1.60333	−16.58	<.0001
EDUCAT	EDUCATIONAL LEVEL	1	3.14220	0.11173	28.12	<.0001
black		1	14.14565	4.24130	3.34	0.0009
hisp		1	19.67001	3.49912	5.62	<.0001
yrscurrjob		1	0.61531	0.02315	26.58	<.0001
blackxeduc		1	−1.39002	0.32659	−4.26	<.0001
hispxeduc		1	−1.75622	0.29772	−5.90	<.0001

Where

Black = a dummy variable for Black

hisp = a dummy variable for Hispanic

yrscurrjob = number of years in the current job

EDUCAT = number of years of education

blackxeduc = Black *x* educat

hispxeduc = hisp *x* educat

For those of you who are starting with this chapter rather than the review in Chapter 1, this is a typical regression output table. The column labeled "Parameter Estimate" lists the *b*'s (coefficients), with standard errors, the *t*-value, and a ***two-tailed*** probability under the null. *If you want a probability for a one-tailed test, divide this probability by two.*

Note that the interaction terms for "blackxeduc" and "hispxeduc," formed by multiplying each dummy variable by education, are each significant. These individual tests show the significance of the difference in the effect of education for Blacks versus Whites and Hispanics versus Whites, respectively. The equation will always show you ***some*** of the differences in impact among groups, but it never can show you all because you cannot see the difference between groups in the equation, in this case, Blacks versus Hispanics.

In this example, there is no overall "main effect" of education in the equation: The coefficient for the effect of education in the equation only shows the *marginal* effect among Whites, the reference group. If you interpret this as the overall effect of education, you are misled. In general, the marginal effect of *X* in an equation including interactions with other variables will always be ***the effect of X when the other variables it interacts with equal zero.***

For reference going forward, following is the SAS program that was used to produce these results:

```
proc reg data=temp simple;
model rtotinc=  educat black hisp yrscurrjob;
model rtotinc=  educat black hisp yrscurrjob blackxeduc hispxeduc;
interaction:    test blackxeduc=hispxeduc=0;
blvshisp:      test blackxeduc-hispxeduc=0;
weight weight;
run;
```

There are generally two phases to a SAS program: a DATA step and a PROC step. The DATA step is used to call in variables from a data set and create new variables or alter the coding of existing variables. This step prepares the variables you use in the analysis. In the appendix to this chapter, we present both the SAS and the STATA code that can be used to generate the variables used in this analysis from the raw data. That code is broadly annotated to indicate the functions of various statements.

What you see above is the PROC step, invoked to run the SAS REG procedure. The "data" keyword states the name of the data set to analyze—here this is a temporary data set created in a previous DATA step called "temp." The "simple" keyword asks for descriptive statistics with the output (not shown). Each SAS statement ends with a semicolon.

MODEL statements specify different regression models to be estimated. The first model here is "nested" in the second model. Note the two TEST statements following the model statements. These statements can be inserted after any specific regression model to conduct many types of post-hoc tests. Here there are two: The first tests the null hypothesis that both of the interaction terms are zero—*this test is equivalent to the R^2 test mentioned above*—and the second is a specific test for the difference in the effect of education (i.e., the slopes) among Blacks versus Hispanics. You use the variable names from the model to refer to the coefficients. The logic of the first test is that if there is no interaction, then all of the interaction terms should be zero. The second test is important. As noted above, the results can only show you the difference in the slopes of groups relative to the reference group *but not with respect to each other.* Since each interaction b is the difference in the effect of education between each group in the equation and Whites, then the slope difference between those groups must be the difference between their coefficients.

In this example, the effect of education among Whites is $b = 3.142$. The effect among Blacks is 1.39 lower than that, and the effect among Hispanics is 1.76 lower than that. Thus the difference in the slopes for Blacks versus Hispanics has to be $-1.39 - (-1.76) = .37$, or $b_5 - b_6$, because their coefficients are differences from the same reference point.

This point can be more formally demonstrated as follows. You can always design tests to compare slopes in different groups once you have worked out the coefficients from the equation that contribute to each slope, as in Table 2.2.

For example, we know the following:

The effect of education among Blacks is $(b_1 + b_5)E$.

The effect of education among Hispanics is $(b_1 + b_6)E$.

The null hypothesis of "no difference" in effect between Blacks and Hispanics is

$$H_0 : b_1 + b_5 = b_1 + b_6$$

which is equal to

$$\begin{aligned}(b_1 + b_5) - (b_1 + b_6) &= 0\\ b_1 + b_5 - b_1 - b_6 &= 0\\ b_5 - b_6 &= 0\end{aligned}$$

This is a simple example, but in the case of three-way interactions, the specific components of the slope in each group are more complex. Table 2.4 shows the output from the "interaction" test in SAS.

An F value of 23.60, $p < .0001$ with degrees of freedom of 2 and 5568, suggests that there is a strong significant difference between the effect of education on income by racial groups.

TABLE 2.4 ⬢ TEST OF THE INTERACTION IN PROC REG

Test interaction Results for Dependent Variable rtotinc				
Source	**DF**	**Mean Square**	***F* Value**	**Pr > F**
Numerator	2	8921.78230	23.60	<.0001
Denominator	5568	378.10527		

However, the test of the differences between the slopes for Blacks versus Hispanics is not significant, as shown in the output by the test in Table 2.5.

In other words, our results imply that Blacks and Hispanics suffer a similar level of disadvantage relative to Whites in terms of returns to education.

TABLE 2.5 ● TEST OF THE DIFFERENCE IN THE EFFECT OF EDUCATION AMONG BLACKS VERSUS HISPANICS

Test blvhisp Results for Dependent Variable rtotinc				
Source	**DF**	**Mean Square**	***F* Value**	**Pr > F**
Numerator	1	264.25945	0.70	0.4032
Denominator	5568	378.10527		

The regression results in Table 2.3 only show us *differences in effects,* not the actual slopes within the groups involved. Table 2.2 shows the manual way to derive the slopes in each group. But there is additional information we may still need. This is often important in specific analyses: Is the effect of *X* still significant in the other groups, or is the effect even significant in the opposite direction?

PROC GLM in SAS can be used to achieve what is done manually in Table 2.2 by using the "estimate" statement. We ran the same model in GLM using the syntax below to derive both an estimate of the within-group slopes and, importantly, their significance.

```
proc glm data=temp;
model rtotinc=  educat black hisp yrscurrjob blackxeduc hispxeduc;
estimate 'blacks' educat +1 blackxeduc +1 ;
estimate 'hispanics' educat +1 hispxeduc +1;
weight weight;
run;
```

You can see that GLM is set up similarly to REG, with additional *estimate* statements. The first *estimate* statement estimates and tests the significance of the slopes among Blacks by setting the *b* for "*educat*" to +1 and the *b* for "*blackxeduc"* to +1. In effect, this adds up the two effects—that is, it operationalizes $(b_1 + b_5)$. The other *estimate* statement does the same for the effect among Hispanics, using a test for $(b_1 + b_6)$. The results are shown in the output from GLM in Table 2.6.

Results show the same slopes as in Table 2.2, plus the fact that each is still significant.

TABLE 2.6 ● POST-HOC TESTS ON EDUCATION SLOPES AMONG BLACKS AND HISPANICS

Parameter	Estimate	Standard Error	*t* Value	Pr > |t|
blacks	1.75218596	0.30723825	5.70	<.0001
hispanics	1.38598037	0.27615313	5.02	<.0001

2.1.2.1 Syntax for a Two-Way Interaction in STATA

We can produce the same multiplicative model in STATA using the *regress* procedure. The dependent variable is listed first, followed by the independent variables. To estimate the combined impact of race and education on respondent's total income, we enter the interaction variables into the equation, along with all lower-order terms, and control for seniority. The model can be weighted using the *pweight* command followed by the designated weight variable (in this case "weight") in square brackets.

```
regress rtotinc black hisp yrscurrjob educat blackxeduc hispxeduc
[pweight=weight]
```

The ***regress*** command can also be shortened to ***reg.*** We can test the significance of the overall interaction in STATA similarly to the statement in SAS, using the following:

```
test blackxeduc=hispxeduc=0

 ( 1)  blackxeduc - hispxeduc = 0
 ( 2)  blackxeduc = 0

       F(  2,  5568) = 23.09
            Prob > F = 0.0000
```

The two-step approach to testing the interaction is noted in the first set of lines, followed by the *F*-value (with designated degrees of freedom) and the probability under the null.

We can also test for the difference in the effect of education among Blacks versus Hispanics similarly to SAS:

```
test blackxeduc-hispxeduc=0

 (1)  blackxeduc - hispxeduc = 0

      F(  1,  5568) = 1.50
           Prob > F = 0.2201
```

We note that the results of these tests are not exactly the same as in SAS, for reasons we explain later (section 2.2.1). However, the essential results are the same: The interaction is significant, and the difference between the slopes for Blacks versus Hispanics is not significant.

2.1.3 A Simpler Example

In many cases, you will want to test the difference in the effect of some X across just two groups, defined by dichotomies such as gender, work status, nativity, or any other distinction important to the issue of generalizing your results.

It is plausible to expect a two-way interaction between education and gender in the prediction of personal income. The same hypothesis applies here as in the case of race: Women may receive fewer returns to education in terms of job income relative to males.

To test this interaction, we estimate the following model:

$$\hat{Y} = a + b_1E + b_2F + b_3S + b_4(E \cdot F)$$

Where F is a dummy variable for Female (= 1 for female, = 0 for male). S is job seniority, as before, and *educxfem* in the output is the interaction between education and female. Note there is only one interaction term to be tested. This means we can test for the significance of this interaction using the t test for that term in the regression—it is equivalent to the F-test discussed earlier when only one variable is added to the model.

Estimating this model leads to the results shown in Table 2.7.

TABLE 2.7 ● A TWO-WAY INTERACTION BETWEEN EDUCATION AND GENDER

Parameter Estimates						
Variable	Label	DF	Parameter Estimate	Standard Error	*t* Value	Pr > |t|
Intercept	Intercept	1	−23.77622	1.62065	−14.67	<.0001
female		1	11.09294	2.57168	4.31	<.0001
yrscurrjob		1	0.54237	0.02274	23.85	<.0001
EDUCAT	EDUCATIONAL LEVEL	1	3.24783	0.11350	28.62	<.0001
educxfem		1	−1.50119	0.18964	−7.92	<.0001

You can see that the interaction is significant, and the resulting estimates are

$$\hat{Y} = -23.77 + 3.25 \cdot E + 11.09 \cdot F + .542 \cdot S - 1.50 \cdot (E \cdot F)$$

What are the subgroup equations for the effect of education that apply to females versus males? To figure this out, substitute 0 for females (signifying males) in the equation and simplify, and then substitute 1 for female into the equation, and simplify.

For males (F = 0), the equation is

$$\begin{aligned}\hat{Y} &= -23.77 + 3.25 \cdot E + 11.09 \cdot 0 + .542 \cdot 13.95 - 1.50 \cdot (E \cdot 0)\\ &= -23.77 + (.542 \cdot 13.95) + 3.25 \cdot E\\ &= -16.21 + 3.25 \cdot E\end{aligned}$$

For females, the equation is

$$\begin{aligned}\hat{Y} &= -23.77 + 3.25 \cdot E + 11.09 \cdot 1 + .542 \cdot 13.95 - 1.50 \cdot (E \cdot 1)\\ &= -23.77 + 11.09 + (.542 \cdot 13.95) + 3.25 \cdot E - 1.50 \cdot E\\ &= -5.12 + (3.25 - 1.50) \cdot E\\ &= -5.12 + 1.75 \cdot E\end{aligned}$$

Obviously, women receive a much smaller return for each year of education compared to males—just over half of what males receive. The question is how this combines with the issue of differential returns to race.

2.2 A THREE-WAY INTERACTION BETWEEN EDUCATION, RACE, AND GENDER

The preceding results suggest that both race and gender modify the impact of education. What happens if you consider both simultaneously—as in "race, class, gender"?

The possibility of a three-way interaction here is suggested by this reasoning: The *degree* to which race dampens the effect of education may itself depend on gender. For example, the difference in impact among White men and White women may be *larger* than the difference between minority group men and women. Some may hypothesize the opposite, as in a double jeopardy hypothesis, in which women from racialized groups are doubly disadvantaged. In either case, we have to estimate a three-way interaction to evaluate either possibility.

The three-way model is:

$$\hat{Y} = a + b_1E + b_2B + b_3H + b_4S + b_5F + b_6(E \cdot B) + b_7(E \cdot H) + b_8(E \cdot F) + b_9(B \cdot F) + b_{10}(H \cdot F) + b_{11}(E \cdot B \cdot F) + b_{12}(E \cdot H \cdot F)$$

where as before $F = 1$ for female and 0 for male. You must include a test for all two-way interactions first, in order to isolate—that is, partition, the three-way effect. This is an important feature of this approach: One only attributes importance to a three-way contingency after allowing for all of the combinations of two-way contingencies.

The previous two-way model would be:

$$\hat{Y} = a + b_1E + b_2B + b_3H + b_4S + b_5F + b_6(E \cdot B) + b_7(E \cdot H) + b_8(E \cdot F) + b_9(B \cdot F) + b_{10}(H \cdot F)$$

Note that the three-way model could only be retained if an *F*-test for the increase in R^2 due to adding b_{11} and b_{12} was significant compared to the model with all two-way terms. A true three-way interaction involves the idea that racial differences in the impact of education do not generalize across gender. A careful consideration of the two-way interactions suggests some alternative interpretations. If education just interacts with race and gender *separately*, this means that there are racial differences in the impact of education that apply equally to both genders *and* that there are gender differences in the effect of education that apply equally across racial groups. That is a very different interpretation than the assumption that the gender difference changes depending on the group and is specific to different groups.

2.2.1 Deriving Education Effects for Selected Groups in the Three-Way Equation

To show how you can derive subgroup slopes in a model with a three-way interaction, we provide the calculations symbolically in Table 2.8. Note now that there are six distinct groups, including every combination of race and gender, and there is a unique slope for the effect of education in each group.

In each case, you plug in the combination of values for race and gender that define a particular subgroup and simplify, as before.

You can approach this way of parsing interactions mechanically: Plug in values, remove terms including 0, collect and simplify terms multiplied by the same variable, reduce the equation to

TABLE 2.8 ⬢ ANALYZING A THREE-WAY INTERACTION

Group	Variables			Equation for Effect of E
	B	H	F	$\hat{Y} = a + b_1E + b_2B + b_3H + b_4S + b_5F + b_6(E \cdot B) + b_7(E \cdot H) + b_8(E \cdot F) + b_9(B \cdot F) + b_{10}(H \cdot F) + b_{11}(E \cdot B \cdot F) + b_{12}(E \cdot H \cdot F)$
White males	0	0	0	$\hat{Y} = a + b_1E + b_2(0) + b_3(0) + b_4(13.98) + b_5(0) + b_6(E \cdot 0) + b_7(E \cdot 0) + b_8(E \cdot 0) + b_9(0 \cdot 0) + b_{10}(0 \cdot 0) + b_{11}(E \cdot 0 \cdot 0) + b_{12}(E \cdot 0 \cdot 0)$ $= (a + b_4(13.98)) + (b_1)E$
White females	0	0	1	$\hat{Y} = a + b_1E + b_2(0) + b_3(0) + b_4(13.98) + b_5(1) + b_6(E \cdot 0) + b_7(E \cdot 0) + b_8(E \cdot 1) + b_9(0 \cdot 1) + b_{10}(0 \cdot 1) + b_{11}(E \cdot 0 \cdot 1) + b_{12}(E \cdot 0 \cdot 1)$ $= (a + b_4(13.98) + b_5) + (b_1 + b_8)E$
Black males	1	0	0	$\hat{Y} = a + b_1E + b_2(1) + b_3(0) + b_4(13.98) + b_5(0) + b_6(E \cdot 1) + b_7(E \cdot 0) + b_8(E \cdot 0) + b_9(1 \cdot 0) + b_{10}(0 \cdot 0) + b_{11}(E \cdot 1 \cdot 0) + b_{12}(E \cdot 0 \cdot 0)$ $= (a + b_2 + b_4(13.98)) + (b_1 + b_6)E$
Black females	1	0	1	$\hat{Y} = a + b_1E + b_2(1) + b_3(0) + b_4(13.98) + b_5(1) + b_6(E \cdot 1) + b_7(E \cdot 0) + b_8(E \cdot 1) + b_9(1 \cdot 1) + b_{10}(0 \cdot 1) + b_{11}(E \cdot 1 \cdot 1) + b_{12}(E \cdot 0 \cdot 1)$ $= (a + b_2 + b_4(13.98) + b_5 + b_9) + (b_1 + b_6 + b_8 + b_{11})E$
Hispanic males	0	1	0	$\hat{Y} = a + b_1E + b_2(0) + b_3(1) + b_4(13.98) + b_5(0) + b_6(E \cdot 0) + b_7(E \cdot 1) + b_8(E \cdot 0) + b_9(0 \cdot 0) + b_{10}(1 \cdot 0) + b_{11}(E \cdot 0 \cdot 0) + b_{12}(E \cdot 1 \cdot 0)$ $= (a + b_3 + b_4(13.98)) + (b_1 + b_7)E$
Hispanic females	0	1	1	$\hat{Y} = a + b_1E + b_2(0) + b_3(1) + b_4(13.98) + b_5(1) + b_6(E \cdot 0) + b_7(E \cdot 1) + b_8(E \cdot 1) + b_9(0 \cdot 1) + b_{10}(1 \cdot 1) + b_{11}(E \cdot 0 \cdot 1) + b_{12}(E \cdot 1 \cdot 1)$ $= (a + b_3 + b_4(13.98) + b_5 + b_{10}) + (b_1 + b_7 + b_8 + b_{12})E$

components of the intercept and components of the effect of X, plug in coefficients to calculate. If there is one thing this calculation shows, it is the fact that it is nearly impossible to look at the results for regression equations including a three-way interaction and interpret them properly. You have to take it apart to understand it. One wonders to what degree this issue plagues hypothesizing and presenting three-way interactions.

There *are* certain things you can see in the results, such as (a) the marginal effect of education is the effect in the combined reference group of White males; (b) the difference in the effect among Blacks and Hispanics, *for males only,* is shown by the two-way interaction between education and race; and (c) the difference in the effect of education for White females is shown by the two-way interaction between education and female. After that, interpretation gets more complex.

You would need to understand these subgroup effects before you could test for differences in slopes across groups. For example, using the equation, to find the difference in effect of education between Black females and Black males:

$$\begin{aligned}&H_0 : b_1 + b_6 + b_8 + b_{11} = b_1 + b_6\\&(b_1 + b_6 + b_8 + b_{11}) - (b_1 + b_6) = 0\\&b_8 + b_{11} = 0\end{aligned}$$

The difference between Black females and Hispanic females is:

$$\begin{aligned}&H_0 : b_1 + b_6 + b_8 + b_{11} = b_1 + b_7 + b_8 + b_{12}\\&(b_1 + b_6 + b_8 + b_{11}) - (b_1 + b_7 + b_8 + b_{12}) = 0\\&b_6 - b_7 + b_{11} - b_{12} = 0\end{aligned}$$

2.2.2 Testing the Three-Way Interaction in a Sequence of Models

To test a three-way interaction, you should include tests for the simpler models first. In this example, this could include six models, from the simple additive "main effects" model to the final three-way model. One ***tests*** these models in reverse order: If the three-way interaction is not significant, then you consider the set of two-way interactions—education x race, education x gender, and race x gender. If one or more of these is significant, you retain and interpret that model. If not, you fall back to a main effects model.

Here is the sequence of models and what each one tests:

$$\hat{Y} = a + b_1E + b_2B + b_3H + b_4S + b_5F \tag{1}$$

1. Model 1 tests the main effects of race, gender, seniority, and education only.

$$\hat{Y} = a + b_1E + b_2B + b_3H + b_4S + b_5F + b_6(E \cdot B) + b_7(E \cdot H) \tag{2}$$

2. Model 2 adds a two-way interaction between education and race. Model 1 is nested in Model 2. Model 2 tests this interaction on its own first.

$$\hat{Y} = a + b_1E + b_2B + b_3H + b_4S + b_5F + b_8(E \cdot F) \tag{3}$$

3. Model 3 adds a two-way interaction between education and gender to Model 1, to test it on its own first.

$$\hat{Y} = a + b_1E + b_2B + b_3H + b_4S + b_5F + b_9(B \cdot F) + b_{10}(H \cdot F) \tag{4}$$

4. Model 4 adds a two-way interaction between gender and race to Model 1, to test on its own.

$$\begin{aligned}\hat{Y} = a + b_1E + b_2B + b_3H + b_4S + b_5F + b_6(E \cdot B) + b_7(E \cdot H) + b_8(E \cdot F)\\ + b_9(B \cdot F) + b_{10}(H \cdot F)\end{aligned} \tag{5}$$

5. Model 5 adds all three two-way interaction to Model 1, to test the collective hypothesis of ***any*** two-way interactions involving education, race, and gender. Note that Model 1 is nested in all of the models from 2 through 5.

$$\hat{Y} = a + b_1E + b_2B + b_3H + b_4S + b_5F + b_6(E \cdot B) + b_7(E \cdot H) + b_8(E \cdot F) + b_9(B \cdot F) + b_{10}(H \cdot F) + b_{11}(E \cdot B \cdot F) + b_{12}(E \cdot H \cdot F) \quad (6)$$

6. Model 6 adds the two terms necessary to test the three-way interaction. Model 5 is nested in Model 6. So are the simpler models, but those comparisons are not interesting because they do not isolate specific effects.

If the three-way interaction in Model 6 is not significant and one or more of the two-way interactions in Models 2, 3, and 4 are significant, then one can use Model 5 to figure out which two-way interactions could be retained in the presence of others. However, *all* should be retained to test the three-way interaction.

We do not show results for Models 2 through 5 here, although *all* of the two-way interactions involved here were significant. But those results would be misleading if there is a three-way interaction here, and this interaction is significant. Results from the three-way model are shown in Table 2.9.

TABLE 2.9 ⬢ A THREE-WAY INTERACTION INVOLVING EDUCATION, RACE, AND GENDER

Parameter Estimates

Variable	Label	DF	Parameter Estimate	Standard Error	*t* Value	Pr > |t|
Intercept	Intercept	1	–27.81590	1.94280	–14.32	<.0001
black		1	19.26546	5.36993	3.59	0.0003
hisp		1	22.35430	4.10073	5.45	<.0001
female		1	14.33510	3.07161	4.67	<.0001
yrscurrjob		1	0.52890	0.02280	23.20	<.0001
EDUCAT	EDUCATIONAL LEVEL	1	3.59936	0.13415	26.83	<.0001
blackxeduc		1	–1.90294	0.41763	–4.56	<.0001
hispxeduc		1	–2.16029	0.34772	–6.21	<.0001
educxfem		1	–1.77564	0.22269	–7.97	<.0001
femxblack		1	–19.39456	8.19880	–2.37	0.0180
femxhisp		1	–15.11531	7.14670	–2.12	0.0345
edxfemxblack		1	1.84471	0.62959	2.93	0.0034
edxfemxhisp		1	1.56790	0.60536	2.59	0.0096

Test any3way Results for Dependent Variable Rtotinc

Source	DF	Mean Square	*F* Value	Pr > F
Numerator	2	2413.99144	6.77	0.0012
Denominator	5562	356.59939		

We use the same variable naming conventions as for the earlier two-way example. For example, "edxfemxblack" is a three-way interaction term formed from multiplying these variables: *educat* * *female* * *black*.

The post-hoc tests shows that the three way interaction here is significant, and the individual terms are also significant. When you look at regression results for a three-way model, it is very difficult to "see" the results. You can refer to the calculations on page 61 to derive subgroup slopes by hand.

Looking at the equation here, however, you can see that the effect of education for White males (the reference group) is 3.599 thousand dollars of income per year of education. The net effect among Black males must be: 3.559 – 1.90 (the coefficient for blackxeduc) = 1.66 thousand dollars per year of education. The net effect among White women is 3.559 – 1.78 = 1.78—that is, *half* of the effect among White males.

Note however that the coefficients for the two three-way terms are positive. This is where things get subtle and could be misinterpreted. This result means that the effect of education is not as low as one would expect from the combined effect of being a minority and being female as suggested by the two-way interactions. In fact, being female counteracts some of the negative effect due to minority status, so that instead of a cumulative "double jeopardy" effect due to two disadvantaged statuses, we see a "ceiling effect," where either one counts, but further indicators of disadvantaged status do not add to the effects of the other. In other words, these results argue for a "one is enough" rule, which is a form of intersectionality, but it is not the form most often predicted.

As an example from the equation, the effect among Black females is 3.60 (*EDUCAT)* – 1.77 (*EDUCAT * female*) – 1.90 (*EDUCAT * Black)* + 1.84 (*EDUCAT * Black * female*) = 1.77, almost the same as the slopes for White women and Black men. These terms show what is happening in the final slope: The three-way term has to counteract the implications of the two-way disadvantages due to race and gender to get to the actual slope. It is easy to see the result in a plot, as shown in Figure 2.2.

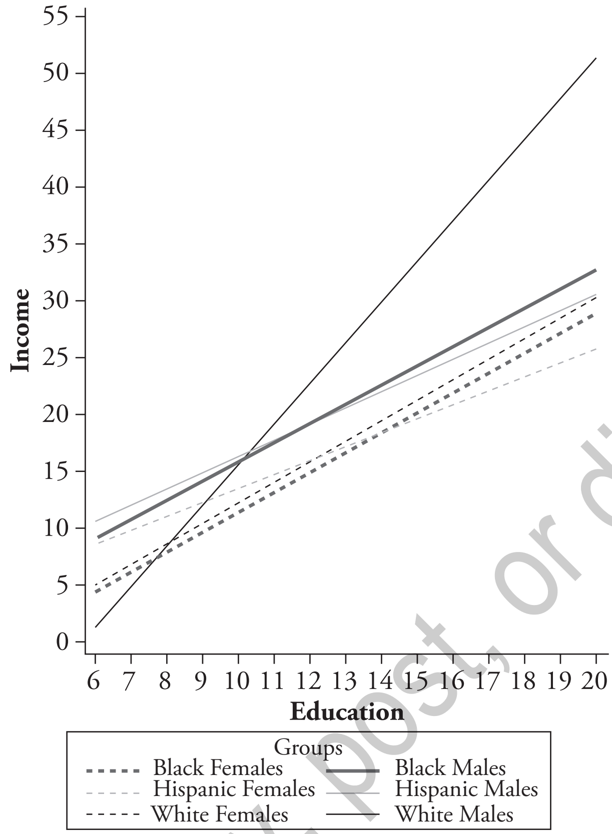
This graph makes the nature of the three-way interaction here quite clear: *Everyone* suffers a "penalty" in the effect of education relative to White males, by a similar amount. The only advantaged group is White males. The former finding at the beginning of this section is misleading and therefore "wrong." It is not that Whites have an advantage, it is that White males specifically—and only White males—have an advantage relative to others.

If you look at the plot closely, it is clear that Black and Hispanic females do not suffer double jeopardy: Their lines are almost parallel to their male counterparts. In effect, from the point of view of gender, this also means that only White females suffer a gender disadvantage. In other groups, minority status trumps gender.

2.2.2.1 Comparing Weighted OLS Regression Results in SAS and STATA

There is one difference between OLS regression results in SAS and STATA that we must underscore. The difference results from the use of a case weight, which was included in our estimation of the previous models. Case—or sample—weights are designed to increase the generalizability of results from a sample to the broader population. Each respondent is assigned a weight that represents the proportion of the population in which their individual characteristics actually occur. These characteristics usually include basic social and demographic features, such as gender, age, marital status, education level, household income, and household size. If the combination of the respondent's characteristics are overrepresented in the sample relative to the population,

FIGURE 2.2 ⬢ A THREE-WAY INTERACTION INVOLVING EDUCATION, RACE, AND GENDER



they are assigned a proportional weight less than 1. If the respondent's combined characteristics are underrepresented in the sample relative to the population, they are assigned a proportional weight greater than 1.

We discuss weights in relation to our SAS versus STATA output because in OLS regression, STATA automatically produces ***robust*** standard errors when using a sample weight in the model statement. This is not the case in SAS. Robust standard errors account for non-normal variances based on the observed data. These calculated errors are often larger than normal standard errors and make statistical significance more difficult to observe.

We present the results for the previously discussed three-way interaction model in STATA to demonstrate this difference in the reported standard errors, compared to the SAS output.

Here is the STATA code for our model:

```
reg rtotinc black hisp female yrscurrjob educat blackxeduc
hispxeduc educxfem femxblack femxhisp edxfemxblack edxfemxhisp
[pweight=weight]
```

TABLE 2.10 A THREE-WAY INTERACTION INVOLVING EDUCATION, RACE, AND GENDER IN STATA

```
Linear regression                                      Number of obs   =   5575
                                                       F( 12,  5562)   =  90.34
                                                       Prob > F        = 0.0000
                                                       R-squared       = 0.2723
                                                       Root MSE        = 19.081
```

rtotinc	Robust Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
black	19.26546	5.696079	3.38	0.001	8.098921	30.432
hisp	22.3543	4.338581	5.15	0.000	13.84899	30.85962
female	14.3351	4.170218	3.44	0.001	6.159847	22.51036
yrscurrjob	.5289019	.0327844	16.13	0.000	.4646316	.5931721
educat	3.599364	.2990628	12.04	0.000	3.013084	4.185644
blackxeduc	-1.902943	.4482052	-4.25	0.000	-2.7816	-1.024285
hispxeduc	-2.160289	.3610002	-5.98	0.000	-2.86799	-1.452587
educxfem	-1.775641	.3227628	-5.50	0.000	-2.408382	-1.1429
femxblack	-19.39456	7.136857	-2.72	0.007	-33.38559	-5.403534
femxhisp	-15.11531	4.929981	-3.07	0.002	-24.77999	-5.450616
edxfemxblack	1.84471	.5527853	3.34	0.001	.761035	2.928385
edxfemxhisp	1.567895	.4145766	3.78	0.000	.7551631	2.380627
_ cons	-27.8159	3.932385	-7.07	0.000	-35.52491	-20.10689

Here is the syntax to estimate the hypothesis test for the three-way interaction:

```
test edxfemxblack=edxfemxhisp=0

 ( 1)  edxfemxblack - edxfemxhisp = 0
 ( 2)  edxfemxblack = 0

       F(  2,  5562) =    8.86
            Prob > F =    0.0001
```

2.2.3 A Digression: Interactions, Intersections, Parsimony, and Complexity

There is a direct connection between what interactions test and what the intersectionality perspective claims. Intersectionality is a perspective with many variants, including anti-categorization. But a prominent version considers the essential co-presence of different sources of inequality as fundamental to understanding the total experience of inequality. The most direct translation of this idea is that there is a three-way interaction between race, class, and gender, an interaction that captures the uniqueness of occupying various configurations of multiple statuses described by those terms. If indeed there is a unique combined effect of being Black, female, and less educated, then the three-way interaction should be significant.

There are important differences under the surface between the quantitative study of interactions and the qualitative study of intersections. In testing an interaction, you are requiring that it is *necessary* to describe a given set of relationships, *over and above the cumulative impact of separate main effects, each of which is not contingent on other statuses.* This distinction is not always clearly made in discussions of intersectionality. On the other hand, some versions of intersectionality are also compatible with the notion of separate main effects of race, class, and gender, but with cumulative impacts. In a sense, the quantitative specification makes the ***theoretical*** distinction between the additive version and the interactive version a foreground issue.

The quantitative emphasis on choosing the most parsimonious model that describes the observed relationships allows for simpler cases than considering all sources of inequality at once. Basically, the claim is that not everything matters everywhere all the time. On the other hand, intersectionality draws our attention to the complexity of the combined effects of race, class, and gender and imagines unique social locations described by combinations of these statuses.

What we learn from considering higher-order interactions is that quantitative approaches can also incorporate considerable complexity. What we learn from intersectionality is that the importance of capturing complexity may at times be more important than the need for parsimony.

2.3 INTERACTIONS INVOLVING CONTINUOUS VARIABLES

You can also have interactions between continuous variables, as well as between categorical variables (next section). Sometimes these combinations of effects are seen as "different," but it is important to emphasize that ***the principles of interpretation developed in the previous section apply in the same way to all interactions.***

Those general principles involve three steps: (1) Define one variable in the interaction as focal—this is the variable whose effect you want to analyze; (2) give values to the variable(s) that define conditions under which the effect of the focal variable changes; (3) resolve the equation into subgroup equations that show the difference in the effect of the focal variable under varying conditions.

There ***are*** some specifics to dealing with interactions involving continuous variables that also have to be taken into account.

2.3.1 Interpreting an Interaction with Two Continuous Variables

Suppose you are considering an interaction between education and age, two continuous variables—for example, to study the possibility of cohort changes in the impact of education. Let us suppose that you are interested in demonstrating that the effect of education on income has declined over time. We would have to use a complicated approach to this question involving different cohorts at the same age, in different studies, but here we will take a simple approach.

The approach with continuous variables is to choose appropriately contrasting values of some variable Z, the other continuous variable in the interaction, to calculate the changing effect of the focal X, for example, at -1 and +1 standard deviations from the mean of Z. Suppose that Z = age. In the NSFH data used throughout this chapter, the mean at Wave 1 is 38, and the standard deviation is (about) 12 years. Using these values, one could calculate the effect of education at ages representing -1 SD (38 - 12 = 26) and +1 SD (38 + 12 = 50). There is nothing sacred about choosing these values: This is one convention among many. For example, some use the 25th and 75th percentiles on age. The values you use also depend on the way in which the continuous variable is coded. For example, if age was centered around its mean, the mean is then 0. In that case, you could use -12 and +12.

The overall equation with A = age and a control for seniority in current job (S) is:

$$\hat{Y} = a + b_1 E + b_2 A + b_3 S + b_4 (E \cdot A)$$

At -1 *SD* of age and a mean years in current job = 14,

$$\hat{Y} = a + b_1 E + b_2(26) + b_3(14) + b_4(E \cdot 26) = (a + 26b_2 + 14b_3) + (b_1 + 26 \cdot b_4)E$$

And at +1 *SD* on age,

$$\hat{Y} = a + b_1 E + b_2(50) + b_3(14) + b_4(E \cdot 50) = (a + 50b_2 + 14b_3) + (b_1 + 50 \cdot b_4)E$$

When you estimate this interaction in the NSFH data, you get this result for the overall equation:

$$\hat{Y} = 1.18 + .836E - .535A + .559S + .0466(E \cdot A)$$

The results in Table 2.11 show the interaction between age and education (*agexeduc)* is significant.

TABLE 2.11 ● AN INTERACTION BETWEEN TWO CONTINUOUS VARIABLES

Parameter Estimates						
Variable	Label	DF	Parameter Estimate	Standard Error	*t* Value	Pr > |t|
Intercept	Intercept	1	1.18109	4.18583	0.28	0.7778
age		1	−0.53503	0.09876	−5.42	<.0001
yrscurrjob		1	0.55880	0.03436	16.27	<.0001
EDUCAT	EDUCATIONAL LEVEL	1	0.83582	0.31910	2.62	0.0088
agexeduc		1	0.04661	0.00738	6.32	<.0001

If we work out the net effects of education at two ages—26 and 50—we get the following results, using the substitution of A = 26 and then A = 50 in the equation above:

$$\hat{Y} = 1.18 + .836E - .535(26) + .559(14) + .0466(E \cdot 26)$$
$$\hat{Y} = 1.18 - .535(26) + .559(14) + (.836 + (.0466 \cdot 26))E$$
$$= -4.904 + 2.05E$$

That is, at 26 years old—among the young—the effect of a one-year increase in education is to increase income by just over two thousand dollars a year—given that income is coded in thousands of dollars. At age 50, the net equation for the effect of education is

$$\hat{Y} = 1.18 + .836E - .535(50) + .559(14) + .0466(E \cdot 50)$$
$$\hat{Y} = 1.18 - .535(50) + .559(14) + (.836 + (.0466 \cdot 50)E$$
$$= -17.74 + 3.17E$$

So at age 50, the effect of a one-year increase in education has increased to over three thousand dollars a year.

Whether this is the natural effect of differences in early income multiplying with age or actual cohort differences cannot be determined here. But the mechanics of figuring out the interaction are not affected by this interpretive issue.

2.4 INTERACTIONS BETWEEN CATEGORICAL VARIABLES: THE "*N*-WAY" ANALYSIS OF VARIANCE

In this case, we have two categorical variables. This is a more prevalent case in some disciplines, such as psychology, where experimental designs are prevalent.

You can proceed in the same way in this case as well. What *appears* to be unusual here is that all of the variables in the interaction are categorical, and thus it may seem strange to talk about the "effect" of a variable. But it is done all of the time, as long as you remember that the "effect" of a dummy variable is the mean difference on Y of two groups.

Interactions between two categorical variables are often part of what is called the "two-way analysis of variance." Basically, there is nothing unique about this term, since it is a method used to interpret the effects of two variables as either two additive "main" effects or an interaction. If there are three variables involved, we have a three-way analysis of variance.

Suppose you are interested in studying the distribution of a sense of powerlessness across two categorical variables: marital status and employment status. The hypothesis may be that the impact of unemployment on a sense of powerlessness is much higher in unpartnered marital statuses.

If there is a two-way interaction, it is your choice—depending on your analytical goals—as to which variable is the focal variable. If the point of your analysis is that the meaning of unemployment varies depending on social capital and one of your tests of that idea involves using an interaction with marital status, then you make unemployment focal.

Many people use interactions between categorical variables to derive group mean differences on Y for all groups. This is fine, but it also does not express how one variable changes the impact of the other variable succinctly. Sticking to the logic of an "effect of X" does respect the nature of the interaction.

2.4.1 An Example: A Two-Way ANOVA (Analysis of Variance)

In this case, we have two categorical independent variables, marital status and unemployment. The N is assumed to be 200. The dependent variable here is sense of powerlessness—that is, the percentage of events in your life you perceive as beyond your personal control.

The basic hypothesis to be tested is that the implications of nonemployment for sense of powerlessness will vary across marital statuses and will have a reduced effect among the married, since there is a partner available who may also work.

For simplicity, marital status here has three categories: married, single, and divorced/separated. We develop two dummy variables (***S*** for single and ***D*** for divorced) for marital status, with married as the reference category, as follows.

	S	D
Married	0	0
Single	1	0
Divorced / separated	0	1

Unemployment has two categories; therefore there is just one dummy variable (***U***) for unemployment.

Strictly speaking, in all of the cases we explore in this chapter, you could say that the logic of the analysis is to find the most parsimonious model and yet the most effective model in predicting *Y*.

	U
Working	0
Unemployed	1

We predict an interaction, but if it is not significant, we should interpret the two effects of unemployment and marital status as additive and therefore independent of each other.

The general procedure follows earlier examples:

1. Estimate a model with "additive" effects (main effects) only:

$$\hat{Y} = a + b_1 S + b_2 D + b_3 U$$

$$\hat{Y} = 20 + 6S + 16D + 20U$$

$$R^2 = .20$$

2. Add all possible two-way interaction terms to test for a two-way interaction:

$$\hat{Y} = a + b_1 S + b_2 D + b_3 U + b_4 (S \cdot U) + b_5 (D \cdot U)$$

$$\hat{Y} = 20 + 6S + 12D + 16U + 2(S \cdot U) + 18(D \cdot U)$$

$$R^2 = .34$$

Results here are invented—not based on actual data.

The interaction terms stand for the possibility that mean differences for groups on one variable do *not* generalize across categories of the other group variable. To test whether there are any two-way interactions, conduct an *F*-test for Model (2) versus Model (1).

$$F = \frac{R^2_{(2)} - R^2_{(1)} / k_2 - k_1}{1 - R^2_{(2)} / N - k_2 - 1} \text{ with } k_2 - k_1 \text{ and } N - k_2 - 1 \text{ df}$$

$$= \frac{.34\text{-}.20/2}{1\text{-}.34/200\text{-}5\text{-}1} = 20.58 \text{ p} < .0001$$

This means there *are* significant two-way interactions. So Model (2) is appropriate and should be retained. Further, this means that Model (1) is wrong and should *not* be interpreted.

2.4.2 Resolving the Equation to Show the Effect of One Variable

You can still use the method outlined in the previous sections to analyze this interaction. In fact, in psychology and in many experimental literatures, this type of interaction is what is *typically* seen as an interaction. It is helpful again to focus on the effect of one variable and show how it changes across *categories* of the other variable.

Following the logic above, assume you are discussing the effects of unemployment on sense of powerlessness. In this context, you want to show how this effect changes across marital statuses. We use the overall equation results above to substitute values for different marital status groups into the equation and then resolve the equation to show the effect of unemployment.

This process results in three equations showing the effect of unemployment within the three marital statuses. We can see immediately from the results that the effect of unemployment changes most clearly among the divorced/separated. The effect of unemployment on increasing a sense of powerlessness is much stronger among the divorced/separated, indicating the specific joint consequences of being unemployed and also divorced/separated.

TABLE 2.12 ● RESOLVING THE EFFECT OF UNEMPLOYMENT IN DIFFERENT MARITAL STATUS GROUPS

Marital Status	Dummy Coding		Equation	Subgroup Effect
	S	**D**	$\hat{Y} = a + b_1 S + b_2 D + b_3 U + b_4 (S \cdot U) + b_5 (D \cdot U)$	
Married	0	0	$\hat{Y} = a + b_1 0 + b_2 0 + b_3 U + b_4 (0 \cdot U) + b_5 (0 \cdot U)$ $\hat{Y} = a + b_3 U$	$\hat{Y} = 20 + 16U$
Single	1	0	$\hat{Y} = a + b_1 1 + b_2 0 + b_3 U + b_4 (1 \cdot U) + b_5 (0 \cdot U)$ $\hat{Y} = a + b_1 + b_3 U + b_4 U$ $\hat{Y} = (a + b_1) + (b_3 + b_4) U$	$\hat{Y} = (20 + 6) + (16 + 2)U$ $= 26 + 18U$
Div/Sep	0	1	$\hat{Y} = a + b_1 0 + b_2 1 + b_3 U + b_4 (0 \cdot U) + b_5 (1 \cdot U)$ $\hat{Y} = a + b_2 + b_3 U + b_5 U$ $\hat{Y} = (a + b_2) + (b_3 + b_5) U$	$\hat{Y} = (20 + 12) + (16 + 18)U$ $= 32 + 34U$

2.4.3 Looking More Closely at the Concept of Interaction

We can go one step further in analyzing this equation to reveal exactly what is going on in this interaction, how it is expressed by the regression equation, and how an interaction indicates a specific departure from additivity of effects.

It is important to understand this because ***it is difficult to imagine how interactions are unique relative to the accumulation of multiple independent additive effects***. They *are* different, and it is very important in theoretical terms to make this distinction.

In Table 2.13, we show the predicted Y means for each of the six groups in the equation formed by the consideration of both marital status (three categories) and unemployment (two categories). Unlike before, we are plugging in values for ***all*** variables here, to get the predicted mean level of powerlessness for each group.

TABLE 2.13 ● FIGURING OUT THE MEANS IN ALL GROUPS

Group	S	D	U	Equation: $Y = a + b_1 S + b_2 D + b_3 U + b_4 (S \cdot U) + b_5 (D \cdot U)$
Married, working	0	0	0	$Y = a + b_1(0) + b_2(0) + b_3(0) + b_4(0) + b_5(0) = a = 20$
Married, unemployed	0	0	1	$Y = a + b_1(0) + b_2(0) + b_3(1) + b_4(0) + b_5(0) = a + b_3 = 20 + 16 = 36$
Single, working	1	0	0	$Y = a + b_1(1) + b_2(0) + b_3(0) + b_4(0) + b_5(0) = a + b_1 = 20 + 6 = 26$
Single, unemployed	1	0	1	$Y = a + b_1(1) + b_2(0) + b_3(1) + b_4(1) + b_5(0) = a + b_1 + b_3 + b_4 = 20 + 6 + 16 + 2 = 44$
Divorced/separated, working	0	1	0	$Y = a + b_1(0) + b_2(1) + b_3(0) + b_4(0) + b_5(0) = a + b_2 = 20 + 12 = 32$
Divorced/separated, unemployed	0	1	1	$Y = a + b_1(0) + b_2(1) + b_3(1) + b_4(0) + b_5(1) = a + b_2 + b_3 + b_5 = 20 + 12 + 16 + 18 = 66$

TABLE 2.14 ● TABLE OF MEANS AND COEFFICIENTS

Means	Marital Status		
Unemployment	**Married**	**Single**	**Div./Sep.**
Working	20	26	32
Unemployed	36	44	66

Coefficients Involved in Each Mean	Marital Status		
Unemployment	**Married**	**Single**	**Div./Sep.**
Working	a	$a + b_1$	$a + b_2$
Unemployed	$a + b_3$	$a + b_1 + b_3 + b_4$	$a + b_2 + b_3 + b_5$

Table 2.14 shows the so-called "cell means" for all group combinations. Note from the equation that the *b*'s in general do *not* stand for differences between overall means across groups. You have to derive which *b*'s are involved in the mean of each group.

2.4.3.1 Interpretation of Interaction Term Coefficients

The coefficients b_4 and b_5 show the degree to which mean differences involving the reference categories do not generalize across other levels. For example, if the mean for "div./sep., unemployed" was an additive function of being divorced / separated **plus** being unemployed, then the mean in this cell would be $a + b_2 + b_3$.

Thus, b_5 stands for the degree of departure from additivity. In this case $b_5 = 18$, which means that the actual mean in this cell (66) is 18 points higher than what is predicted by the additive model.

$$\text{That is, } a + b_2 + b_3 = 20 + 12 + 16 = 48.$$

$$b_5 = 18 \text{ is the amount you need to get to 66,}$$

$$\text{that is, } 48 + 18 = 66.$$

The interaction indicates that the specific *combination* of being divorced or separated *and* unemployed results in much higher feelings of powerlessness than would be expected from the combined increases in powerlessness for the divorced or separated when working and from unemployment when married.

2.4.4 Testing Differences Between Group Means

When the *F*-test for an interaction is significant, you may want to know in which groups the interaction occurs. The *F*-test only tells you differences do not generalize across all groups on the other variable but not which groups differ from each other.

You can use the same approach here as for interactions involving continuous variables to conduct tests. Referring to the results for the interaction model, note the equation tests for differences between the following:

- single, working versus married, working (b_1)
- div./sep., working versus married, working (b_2)
- unemployed, married versus working, married (b_3)

To test for differences between other specific groups in the equation, use the coefficients involved to construct tests for differences in effects:

- For div. / sep., unemployed versus div. / sep., working

$$\begin{aligned} H_0\text{: } (a + b_2 + b_3 + b_5) - (a + b_2) &= 0 \\ b_3 + b_5 &= 0 \end{aligned}$$

- For div. / sep., unemployed versus married, unemployed

$$\begin{aligned} H_0\text{: } (a + b_2 + b_3 + b_5) - (a + b_3) &= 0 \\ b_2 + b_5 &= 0 \end{aligned}$$

You can also test for *combinations* of group differences. For example,

(div./sep., unemployed – div./sep., working) – (single, unemployed – single, working) = 0

$$= ((a + b_2 + b_3 + b_5) - (a + b_2)) - ((a + b_1 + b_3 + b_4) - (a + b_1)) = 0$$
$$= (b_3 + b_5) - (b_3 + b_4) = 0$$
$$= b_3 + b_5 - b_3 - b_4 = 0$$
$$= b_5 - b_4 = 0$$

This is a test for the specific location of the interaction. If b_5 is greater than b_4, we know that the effect of unemployment among the divorced / separated is greater than among the single. If we also see that the single do not differ from the married, then we can locate exactly which group is different from the others.

You can also test specific effects of unemployment *within marital status groups* for significance. This is not a test of the difference between groups but a test of the effect of the focal variable within groups. For example, to test the significance of the effect of unemployment among the single, test

$$(b_3 + b_4) = 0$$

And to test the significance of the effect of unemployment among the divorced / separated, test

$$(b_3 + b_5) = 0$$

2.4.5 Three-Way Interactions with Categorical Variables (Three-Way ANOVA)

Studying three-way interactions in the case of categorical variables follows the same logic as with other combinations of types of variables (see section 2.2.2). It is worth emphasizing three fundamental issues in the model-building verses model-testing logic of testing interactions:

1. Combinations of two-way interactions have to form the foundation of testing a full three-way interaction because the theoretical interpretation of these two cases is very different.
2. Model-building proceeds from the simplest additive model to the most complex three-way model, but model-testing proceeds in the reverse, starting with the most complex model.
3. The approach used in this chapter makes distinctions concerning the way in which the effects of variables combine that are not clearly articulated in theoretical models promoting the idea of joint effects.

2.5 CAUTIONS IN STUDYING INTERACTIONS

There are a number of cautions one should take into account in estimating interactions. Here we discuss four that may be important.

2.5.0.1 Multicollinearity

A basic problem in interaction models is that the product terms are made up from other variables in the model, thus introducing positive correlations among independent variables. Transformations of *x* reduce the correlations between main effects and their interactions.

One simple way to reduce multicollinearity is to use "centered" *x*'s—that is, subtracting means from raw *x* scores—that is, $x_1 = X_1 - \bar{X}_1$. In the earlier example discussing the two-way interaction between education and age, we pointed out that in that sample, the mean age was 38. Centering age here means that we subtract 38 from each individual *X* score. This makes the mean of age = 0, and the resulting deviation scores in age are negative values below that (e.g., –1 S.D. = –12), or positive values above that (+1 SD = +12).

2.5.0.2 The Issue of a "Main Effect" in the Presence of Interactions

In general, the safe approach is to realize that an interaction stands for the fact that there is no main effect, and thus you should interpret only the separate subgroup effects. But some do report an *averaged* main effect in the case of an interaction, standing for either an averaged effect across groups or an averaged effect across levels of a continuous variable.

Suppose you are considering an interaction between two ***continuous*** *variables*, such as

$$\hat{Y} = a + b_1 X_1 + b_2 X_2 + b_3 (X_1 \cdot X_2)$$

If you center X_1 and X_2 by subtracting their mean values, as above, thus scaling each so their mean = 0, then, by definition, the effect of X_1 is its effect at the mean of X_2, and the effect of X_2 is its effect at the mean of X_1.

A common mistake often made in models with interactions is that the marginal effect of the focal *X* in the equation with the interaction is still the main effect—it is not. It is only the effect of *X* under the condition that the variable it interacts with equals 0—which is the reference group for dummy variables or the zero point on a continuous scale.

In general, we do *not* advocate presentation of "main" effects in the presence of an interaction. The concept of an averaged main effect also denotes the fact that is hiding important variation in effects across groups or across levels of a continuous variable. We suggest this variation should be in the foreground.

2.5.0.3 Problems with Standardized Solutions

One cannot and should not interpret the standardized coefficients in a model with interactions. The model we would want to assess would be

$$\hat{Y} = a + b_1 Z_1 + b_2 Z_2 + b_3 (Z_1 \cdot Z_2)$$

$$\text{where } Z_1 = X_1 \text{ standardized}$$

$$Z_2 = X_2 \text{ standardized}$$

But in standard computer programs, the variables *as a whole* are standardized so that the product term is $(X_1 \cdot X_2)$ standardized, which is wrong. To get to the correct interpretation, standardize *X*s ***before analysis*** and then interpret the unstandardized results, which are, in effect, standardized variables.

2.5.0.4 Number of Post-hoc Tests

You should practically limit the number of post-hoc tests you conduct to minimize the cumulative problem of committing a Type I error—assuming significance when the real value in the population is "no difference." A reasonable approach is to only test the necessary and essential contrasts for interpretation of the results. Often, the tests you should conduct are suggested by the pattern of the results. Concentrate on tests that establish whether groups differing from the reference group are equal to or different from each other and whether they form subgroups.

There are many methods available for controlling Type I error in post-hoc tests, but many of these methods apply mainly to uncorrelated independent variables in experiments. Our advice here is simple: Only investigate the specific differences in the slopes among groups *after* an overall significant *F*-test for the interaction.

2.6 PUBLISHED EXAMPLES

Interactions are ubiquitous in published research, primarily because they express a fundamental hypothesis of interest across a wide range of research questions: Is this experience shared or distinct? Does this occur in only one group rather than in all groups? Does this generalize to very different countries? Is this still true now, even if it was true then? What activates or de-activates this effect?

In this section, we consider three published examples of the use of interactions to address specific research questions. In each case, we emphasize the role of interactions and their interpretation in achieving the purposes of the research questions in the article.

2.6.1 The Gender-Specific Effect of Marriage

There is a large literature on the gender difference in the effect of marriage on well-being. This is an issue that has had prominence in public discourse for over forty years, with the standard conclusion that men benefit more from marriage than women.

Hall (1999) conducted a meta-analysis of this literature, based on 213 independent parameter estimates derived from 78 studies done from the 1930s up to the 1990s. To give some specificity to the issue being assessed in this research, Table 2.15 shows some *hypothetical* results for this issue, based on mean levels of depression in four groups: never-married males, never-married females, married males, and married females.

Hall points out that we cannot understand this effect properly by using what she calls a ***sequential contrast*** approach, represented in Table 2.15. This approach might utilize *t* test differences among never marrieds (i.e., unmarried women vs. unmarried men), *followed by t* test differences among marrieds (i.e., married women vs. married men) to build an argument for a gender-specific effect. In fact, a considerable portion of the literature in this area has taken this approach. The sequential contrast approach in Table 2.15 suggests that women are more depressed by 1 point among the never married, and that this gender difference is nonsignificant. Among the married, however, this difference increases to 2 points and is significant, suggesting that women are only more depressed than men when married. The problem with these tests is that they do not assess the hypothesis at issue directly; rather, they really only reflect the respective within-role differences and thus cannot be used to infer a gender difference in the well-being *gain* due to marriage.

The hypothesis of a gender-specific effect of marriage requires a single assessment of the female difference for the married versus never married minus the corresponding male difference, in other words, a ***differential gain*** effect requiring estimation of an interaction effect between gender and marital status. Table 2.15 represents how this assessment would work. The mental health gain for women is measured by the average reduction in depression, equal to 3 points. The reduction among men is 4 points, resulting in a differential gain among women equal to –1. That is, they have gained one less point. However, treated as a test of differences in gain—which *is* the appropriate test here—the difference in the difference may not be significant, even though significant gender differences in depression only emerged among the married. In other words, there is no evidence here of a gender-specific effect, even though the sequence of t test differences implies there is.

TABLE 2.15a ⬢ THE GENDER SPECIFIC EFFECT OF MARRIAGE RE-CONSIDERED: THE "SEQUENTIAL CONTRAST" VERSUS THE "DIFFERENTIAL GAIN" APPROACH

	Rates of Depression	
	Male	**Female**
Never Married	17	18
Married	13	15

Sequential Contrast Approach

Among the never married, women have slightly higher rates: 18 − 17 = +1 ($p > .05$).

Among the married, women have significantly higher rates: 15 − 13 = +2 ($p < .05$).

Differential Gain Approach

TABLE 2.15b ⬢ CALCULATIONS IN THE DIFFERENTIAL GAIN APPROACH

Gain for Women	Minus	Gain for Men	Differential Gain
(15 – 18)	–	(13 – 17)	
–3	–	–4	= +1 ($p > .05$)

The same problem occurs in analyses in which men and women are studied separately, even if marital gain is the focus. For example, one can study the effect of getting married among men and women separately and compare the significance versus nonsignificance of the effect of marriage in the two genders. This is still a sequential contrast approach. It studies a pattern of results, but it does not test the fundamental hypothesis.

In 2002, Robin Simon published an article in the American Journal of Sociology entitled "Revisiting the Relationships among Gender, Marital Status, and Mental Health." In that article, using the longitudinal component of the National Survey of Families and Households, she assesses (among other things) the gender-specific effects of entering marriage between Waves 1 and 2. This is studied as a test of the interaction between entry into marriage and gender. Results are shown in Table 2.16.

Simon tests the effects of entering marriage from three different nonmarried statuses: never married, divorced / separated, and widowed. Models 2 and 4 show the results of estimating interactions between gender and marital entry for all three cases. Despite the accumulated reputation of a gender-differential effect of marriage specifically applied to the transition from never married to married status, there is no evidence here of an interaction between gender and entering marriage. In fact, none of the interactions for changes in depression

TABLE 2.16 ● UNSTANDARDIZED COEFFICIENTS FROM REGRESSIONS OF DEPRESSION AND ALCOHOL ABUSE ON GENDER AND MARITAL GAIN AMONG RESPONDENTS WHO WERE UNMARRIED AT T1

	Depression		Alcohol Abuse	
	Model 1[a]	Model 2[a]	Model 3[a]	Model 4[a]
Female (0, 1)	2.10*** (.58)	2.41*** (.71)	–1.28*** (.11)	–1.42*** (1.33)
T1 depression/alcohol abuse[b]	.29*** (.01)	.29*** (.01)	2.62*** (.33)	2.61*** (.33)
Marital gain from previously never married	–3.88*** (.86)	–3.38*** (1.16)	–.24 (.16)	–.34 (.22)
Marital gain from previously separated/divorced	–2.65** (.86)	–2.08 (1.34)	–.28 (.16)	–.67** (.25)
Marital gain from previously wid- owed	–3.80 (2.38)	–3.22 (3.90)	–.22 (.45)	–1.05 (.74)
Female x marital gain from previously never married	. . .	–.98 (1.54)	. . .	.21 (.29)
Female x marital gain from previously separated/ divorced	. . .	–.92 (1.67)	. . .	.64* (.32)
Female x marital gain from previously widowed	. . .	–.87 (4.92)	. . .	1.30 (.93)
Adjusted R^2	.18	.18	.09	.09

Source: Simon, R. (2002). Revisiting the relationships among gender, marital status, and mental health. *American Journal of Sociology, 107*(4), 1082. doi:10.1086/339225

Note: Numbers in parentheses are SEs. The stably unmarried are the reference category. N = 3,407.

a. Each model controls for sociodemographic variables including age, race, education, and household income, as well as respondent's employment and parental status at T2.

b. Respondent's level of depression at T1 is included in the depression models and whether they reported alcohol problems at T1 is included in the alcohol abuse models.

* P < .05, two-tailed tests.

** P < .01.

*** P < .001.

are significant. There is one significant interaction for alcohol abuse, suggesting that men who *re*-marrry after a prior divorce do experience a greater reduction in alcohol problems. This is indicated by a significant –.67 effect among men (the group coded 0 on the gender dummy variable), counteracted by a .64 weaker effect among women, indicated by the interaction. In effect, this means the net effect among women was –.67 + .64 = –.03, in other words, no change at all. Most of the prior research on this issue centers on emotional well-being outcomes and entry into first marriage, and so the finding of no interaction for depression suggests a very different picture than what was widely assumed in the decades before this article.

This is a case where the "intuitive" approach to the issue does not actually test the hypothesis. What we learn using the interaction, applied longitudinally to the same people entering marriage over time, is that the widely assumed male advantage in first marriages does not apply.

2.6.2 Two Distinct Issues in an Interaction: Race, Gender, and Chains of Disadvantage

There is a widespread tendency in assessing interactions to present the interaction merely as a difference in the effect of some variable across groups. This leads to tables in publications where we "see" the interaction as the interaction coefficient from the estimated model expressing this difference.

But there is more one can and should extract from an interaction in many applications. The difference coefficient in the interaction only expresses a differences in slopes *but not the size of the slope within groups.* This latter issue may be fundamental to the interpretation of the interaction, beyond the issue of an effect difference. For example, imagine this general example. Suppose we find an interaction between race and sex, where each is dummy coded into two groups, Black versus White and Women versus Men. The interaction shows a baseline effect of Black on a sense of powerlessness equal to $b = .5$. The interaction with female is *-.45.* This means that there is an effect of race on a sense of powerlessness among men, but not among women. The .5 effect among men is reduced to *.5 - .45* =.05 race difference among women, which we imagine is zero. This leads to a specific interpretation, including the fact that Black–White differences occur only among men.

If we change the baseline coefficient for race here to .2, instead of .5 but maintain the difference in effects denoted by the interaction, we get a very different interpretation. We can still imagine here that among men, Blacks have a higher sense of powerlessness. However, among women the race difference is .2 - .45 = -.25. In this case, the pattern suggests that Black women have a ***lower*** sense of powerlessness relative to White women, even though the interaction coefficient is the same. This obviously leads to a very different interpretation because in this case, the race difference is the opposite depending on gender and not just an issue of presence / absence. Now we should ask why White women feel more powerless than Black women.

Many, if not most, articles fail to report these within group slope differences as a regular part of the interpretation of the interaction. A significant exception occurs in a recent article by Debra Umberson and colleagues entitled "Race, Gender, and Chains of Disadvantage: Childhood Adversity, Social Relationships, and Health." This article explores, in part, the gender-specific race consequences for exposure to "chains of disadvantage" denoted first by the experience of childhood adversity and compounded by the transfer of childhood adversity into relationship strain in adulthood (Umberson et al., 2014).

In the article, Umberson et al. (2014) suggest reasoning for a race by gender interaction this way:

> This race effect is likely to be stronger among men than women because of gendered relationship processes. Gendered systems foster expressions of masculinity (e,g., self-sufficiency, independence, strength, controlled expression of emotions) that may interfere with close relationships (Connell & Messersehmidt, 2005; Courtcnay, 2000; Williams 2003). Indeed, studies show that compared with women, men are less likely to have close and confiding relationships, to share their feelings with others, and to provide and seek emotional support from others (Rosenfield, Lcnnon, and White 2005; Taylor et al. 2000; Umberson et al. 1996). Scholars suggest that these gendered processes may be more exaggerated for Black men compared with white men because many Black men lack access to other ways of practicing masculinity, such as occupational and economic success. (Connell & Messersehmidt, 2005, p. 23)

Umberson et al. (2014) show race by gender interactions for both childhood adversity and relationship strain in adulthood in Table 2.17.

TABLE 2.17 ● HYPOTHESIS 1: ORDINARY LEAST-SQUARES MODELS ESTIMATING RACE AND GENDER DIFFERENCES IN STRESS OVER THE LIFE COURSE AND ADULT RELATIONSHIP STRAIN AND SUPPORT (*N* = 3,477).

	Childhood Adversity		**Adult Stress Burden**		**Relationship Strain in Adulthood**		**Relationship Support in Adulthood**	
	Wave 1	**Wave 1**	**Wave 1**	**Wave 2**	**Wave 1**	**Wave 2**	**Wave 1**	**Wave 2**
Variables	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**
Female	.115* (.045)	.181** (.053)	.062 (.037)	.060 (.040)	–.001 (.024)	–.042* (.023)	.120*** (.024)	.098*** (.026)
Black	.061 (.046)	.219** (.080)	.161*** (.041)	.233*** (.046)	.132*** (.035)	.059** (.034)	–.025 (.026)	.050 (.028)
Relationship strain in adulthood (W1)	—	—	—	—	—	.546*** (.016)	—	—
Relationship support in adulthood (W1)	—	—	—	—	—	—	—	.468*** (.018)
Adult stress burden (W1)	—	—	.362*** (.022)	—	—	—	—	—
Female*black	—	–.237* (.098)	—	—	–.100* (.043)	—	—	—
R^2	.02	.04	.16	.22	.12	.41	.03	.32

Source: Umberson, D., Williams, K., Thomas, P. A., Liu, H., & Thomeer, M. B. (2014). Race, gender, and chains of disadvantage: Childhood adversity, social relationships, and health. *Journal of Health and Social Behavior, 55*(1), 27. doi:10.1177/0022146514521426

Note: Age controlled when predicting childhood adversity. Age, income, education, and marital status controlled for all other models. Flags for number of missing relationships are also controlled in models predicting adult relationship strain and support. W1 = Wave 1. Unstandardized coefficients. Standard errors in parentheses.

$*p < .05$, $**p < .01$, $***p < .001$ (two-tailed test).

In both cases, Umberson et al. (2014) make the case that the race effect occurs among men but not among women, by calculating and showing the slopes among women as well. For example, in the case of childhood adversity they say this:

> This interaction term is significant and indicates that black men report significantly more childhood adversity than White men (.219); however, this difference is not significant among women (.219 – .237 = –.018).

The same point is made for the interaction predicting relationship strain, where the net race difference among women is .132 - .100 = .032. Including these within-group differences by gender is important to the overall interpretation because now we know that the race difference observed only occurs among men. We could have observed, for example, a weaker, but still significant effect among women, which leads to a more general race difference interpretation. This result, however, is very much an intersectionality interpretation: The effect only occurs in one group, and generalizations to broader considerations of race per se are misleading.

Later in the same article, Umberson et al. (2014) have the opportunity to present two two-way interactions in the same model. This often causes problems because the interpretation gets subtle. The important point is the careful language that goes with multiple two-way interactions, as opposed to a true three-way interaction. Table 2.18 shows these interactions, predicting relationship strain in adulthood. Panel C of this table has two two-way interactions, one between race and gender and the other between race and childhood adversity, in predicting relationship strain in adulthood at Wave 1 of the American's Changing Lives study.

If we concentrate on race as the focal variable, we could interpret these interactions this way: At *any* level of childhood adversity, there is a race by gender interaction in predicting relationship strain. The Black–White difference among men is nonsignificant (b = .055), but at the same time, there a significantly more negative effect among women. In fact, this effect works out to be .055 -.091 = -.036. Although it is not reported in the article, this is also likely not to be significant. So it is possible to have two within-group effects that each are not significant, but the *difference* between them can be significant—very important.

For both genders equally, there is also a two-way interaction between race and childhood adversity. This interaction says that each additional childhood adversity activates the Black–White difference further by .057. So at adversity = 2, the net effect of Black is .055 + .057*2 = .169. This enhanced impact of childhood adversity, importantly, applies to both genders because this is a two-way interaction.

This article is a good example of specific reasoning matched to interactions presented and interpreted in appropriate detail. Another possible strategy is to "take apart" the interaction, as we have in earlier examples, and show the effects within subgroups separately. This is a matter of choice, but the advantage of this approach is that you can see exactly how other variables influence an

TABLE 2.18 ⬢ HYPOTHESIS 2: ORDINARY LEAST-SQUARES MODELS ESTIMATING THE EFFECT OF CHILDHOOD ADVERSITY AND ADULT STRESS BURDEN ON ADULT RELATIONSHIP STRAIN AND ADULT RELATIONSHIP SUPPORT, BY RACE AND GENDER (N = 3,477).

	Relationship Strain in Adulthood		Relationship Support in Adulthood	
	Wave 1 (1)	**Wave 2 (2)**	**Wave 1 (3)**	**Wave 2 (4)**
Panel A: Base model				
Female	–.007	–.045*	.125***	.098***
	(.024)	(.019)	(.024)	(.025)
Black	.113**	.058**	–.046	.050
	(.034)	(.019)	(.024)	(.026)
Relationship strain in adulthood (W1)	—	.547***	—	—
		(.016)		
Relationship support in adulthood (W1)	—		—	.469***
				(.018)
Female*black	–.102*	—	—	—
	(–.102)			
R^2	.14	.40	.03	.39
Panel B: Control for childhood adversity				
Female	–.014	–.048*	.131***	.101***
	(.024)	(.019)	(.024)	(.026)
Black	.104**	.057**	–.041	.050
	(.034)	(.019)	(.024)	(.026)

(*Continued*)

TABLE 2.18 ⬢ Continued

	Relationship Strain in Adulthood		Relationship Support in Adulthood	
	Wave 1 (1)	Wave 2 (2)	Wave 1 (3)	Wave 2 (4)
Relationship strain in adulthood (W1)	—	.543*** (.016)	—	—
Relationship support in adulthood (W1)	—	—	—	.466*** (.019)
Female*black	−.091* (.043)	—	—	—
Childhood adversity	.039*** (.010)	.025** (.008)	−.066*** (.012)	−.024* (.011)
R^2	.12	.39	.03	.32
Panel C: Interaction of childhood adversity with race				
Female	−.012 (.024)	−.047* (.019)	.130*** (.024)	.099*** (.026)
Black	.055 (.039)	.023 (.024)	−.030 (.033)	.091** (.033)
Relationship strain in adulthood (W1)	—	.541*** (.016)	—	—
Relationship support in adulthood (W1)	—	—	—	.465*** (.018)
Female*black	−.091* (.043)	—	—	—
Childhood adversity	.024 (.011)	.015 (.009)	−.063*** (.014)	−.011 (.013)
Black*childhood adversity	.057** (.022)	.040* (.018)	−.012 (.026)	−.048* (.024)
R^2	.12	.39	.03	.32

Source: Umberson, D., Williams, K., Thomas, P. A., Liu, H., & Thomeer, M. B. (2014). Race, gender, and chains of disadvantage: Childhood adversity, social relationships, and health. *Journal of Health and Social Behavior, 55*(1), 28. doi:10.1177/0022146514521426

Note: All models control for age and number of missing relationships. Panels D and E also control for income, education, and marital status. Unstandardized coefficients. Standard errors in parentheses. W1 = Wave 1.

+*p* = .10, **p* < .05, ***p* < .01, ****p* < .001 (two-tailed test).

effect within each subgroup. It is obvious from Table 2.18 here that the article is only explaining the effect of childhood adversity among Blacks, since the effect of adversity among Whites is not significant. Thus, tracking the effect of early adversity controlling for adult stress burden among Blacks would show the degree to which adult stress explains this effect in this group specifically.

2.6.3 A Three-Way Interaction

Everything gets interpretively more complex when you consider three-way interactions. You can see from our earlier example a basic issue in interpreting three-way interactions: It is difficult to “see” what the three-way terms actually represent because their interpretation depends on the lower-order two-way terms. As a result, presentation of three-way interactions—and the accompanying language—becomes much more difficult. The appropriate language itself is an issue: One has to avoid stating the interaction as separate additive effects, unintentionally, and it is difficult to capture the true nature of the three-way contingency.

A recent article by Jonathan Koltai and Scott Schieman includes an essential three-way interaction as part of the argument ("Job Pressure and SES-Contingent Buffering: Resource Reinforcement, Substitution, or the Stress of Higher Status?" *Journal of Health and Social Behavior*, 2015). This article (in part) studies the effect of job pressure on anxiety, using the 2008 National Study of the Changing Workforce. Usually, job pressure is considered a demand characteristic in the workplace with negative consequences. However, job-related resources may intervene to ameliorate these consequences—this is the "buffering hypothesis" of the job demands-resources model. Job resources, such as autonomy, should help to reduce the consequences of job pressure. Koltai and Schieman insert SES into this model, suggesting that the joint effect of job demands and resources has very different meanings at different levels of SES.

Their argument is that the meaning of job resources may change in higher status jobs—autonomy may not appear to be a resource because higher status jobs involve greater responsibility for workplace outcomes. Table 2 from that article (see Table 2.19) shows three-way interactions between job pressure, job autonomy, and either high education or high income, in predicting anxiety levels (Models 2 and 4). The two interactions are similar in form.

All of the components of the three-way interaction are shown in this table. Notice that none of the two-way terms in this model are significant, for either model. The three-way term, however, suggests that the effect of job pressure on anxiety is enhanced specifically

TABLE 2.19 ⬢ ANXIETY REGRESSED ON JOB PRESSURE, JOB-RELATED RESOURCES, SOCIOECONOMIC STATUS, AND INTERACTIONS (*N* = 3,284)

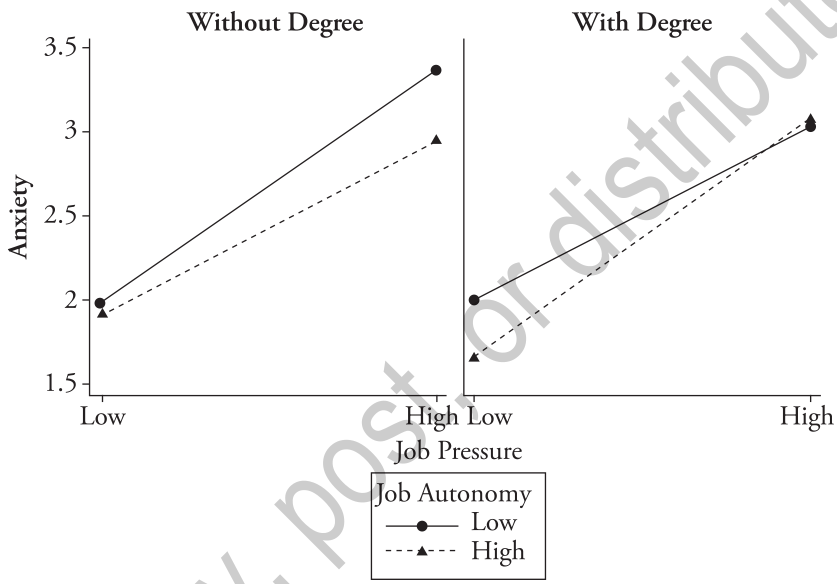
	Model 1	Model 2	Model 3	Model 4	Model 5
Job pressure					
Job pressure	.352***	.341***	.343***	.333***	.336***
Job-related resources					
Job autonomy	–.094***	–.116***	–.096***	–.113***	–.096***
Challenging work	–.126***	–.128***	–.176***	–.127***	–.128**
Socioeconomic status					
High education	–.112**	–.106**	–.118**	–.114**	–.111**
High income	–.089*	–.090*	–.088*	–.081*	–.095*
Interaction terms					
Job Pressure × Job Autonomy	—	–.050	—	–.047	—
Job Pressure × High Education	—	–.018	.007	—	—
Job Autonomy × High Education	—	–.040	—	—	—
Job Pressure × Job Autonomy× High Education	—	.105*	—	—	—
Job Pressure × Challenging Work	—	—	–.073	—	–.067
Challenging Work× High Education	—	—	.099	—	—
Job Pressure × Challenging Work × High Education	—	—	.193**	—	—
Job Pressure × High Income	—	—	—	.024	.017
Job Autonomy × High Income	—	—	—	.027	—
Job Pressure × Job Autonomy × High Income	—	—	—	.094*	—
Challenging Work × High Income	—	—	—	—	–.013
Job Pressure × Challenging Work × High Income	—	—	—	—	.173*
Constant	2.411***	2.403***	2.428***	2.398***	2.415***

Source: Koltai, J. & Schieman, S. (2015). Job pressure and SES-contingent buffering: Resource reinforcement, substitution, or the stress of higher status?" *Journal of Health and Social Behavior, 56*(2), p. 189, Table 2.

among workers with high income or high education and greater job autonomy—the "stress of higher status." But this interaction would be more difficult to interpret if some of the two-way components were also significant.

One could work out a set of subgroup slopes for the effect of job pressure, as we did earlier in the chapter, but Koltai and Schieman use another very effective method: separate graphs of the two-way interaction between job pressure and job autonomy for those without versus with a university degree. Figure 2.3 shows this graph.

FIGURE 2.3 ⬢ DISPLAYING A THREE-WAY INTERACTION AS CONTRASTING TWO-WAY INTERACTIONS.



Source: Koltai, J. & Schieman, S. (2015). Job pressure and SES-contingent buffering: Resource reinforcement, substitution, or the stress of higher status?" *Journal of Health and Social Behavior, 56*(2), p. 190.

The graph shows the positive effect of job pressure on anxiety, in general, but modified by levels of job autonomy. When the respondent has less than a university degree and thus a job that corresponds to this level of qualifications, job autonomy is helpful in reducing the impact of job pressure—as one would generally expect. But when the respondent has a university degree (or more), job autonomy actually increases the effect of job pressure—evidence of the stress of higher status. The graph reveals some interesting issues about the switch in the role of job autonomy: At lower levels of education, it acts as a classic resource moderator, but at higher levels, there is an initial advantage due to job autonomy at low levels of job pressure that disappears as job pressure increases. The acceleration of the effect of job pressure only makes up the difference with those low in job autonomy—it does not actually produce higher levels of anxiety at any point. It also does not produce levels of anxiety higher than the traditionally understood worse-off group here: those with high levels of job pressure, low autonomy, and less education.

What we see here is the advantage of presenting the graph of the three-way interaction: It not only communicates the three-way difference succinctly, it also shows us where the levels of anxiety are across groups, avoiding an over-interpretation of the change in direction of the role of job autonomy. The graph also efficiently illustrates the nature of a three-way interaction, by showing the difference in a two-way interaction at levels of a third variable.

Concluding Words

This chapter has considered interactions in considerably more detail than most of the discussions in the literature on this issue. It is surprising that so little space is given to the interpretation of interactions in expository statistical writing. The issue is that interactions are a natural and ubiquitous consequence of pursuing results completely, of not accepting the presumption of generalizability. We have encouraged the consideration of the intersectionality embodied by interactions for theoretical, practical, and policy reasons. Because interactions constrain our generalizations, they are an extremely important issue in a wide array of questions involving assumptions of personal, institutional, community, or national generalizability.

In this chapter, we have encouraged practices that get more out of the interactions we estimate. There is more than a difference in effects at issue: There is the issue of the existence or reversal in effects across subgroups, and there is the issue of the relative position of groups on the outcome, captured in graphs such as in Koltai and Schieman (2015). These additional pieces of information are essential to the full interpretation of interactions.

Interactions are the first form of departure from the linear additive model we see as standard in many literatures. These models introduce a multiplicative term to represent the possibility of a *condition* in the effect of *X*. In the next chapter, we consider departures from the constraint of linearity and how nonlinear relationships can be represented in these models.

Practice Questions

1. Imagine you want to study the effects of gender and a GPA above 3 on a student's grade in statistics (*Y*, measured out of 100). For gender, you define a dummy variable, *X*1, equal to 1 for females and 0 for males. For GPA, your define a dummy variable, *X*2, equal to 1 if the person has a grade-point average above 3 and 0 if they do not.

   You are interested in the possibility of an interaction between sex and grade-point average in predicting grade in statistics. So you run a regression and find the interaction is significant. The results are

$$Y = 62 - 3X_1 + 5X_2 + 5(X_1 \cdot X_2)$$

   Interpret the interaction by calculating the effect of grade-point average for men and for women.

2. The results in Table 2.A test whether the effect of mother's education on a child's education differs among Blacks and Hispanics relative to Whites. To test this idea, interactions were tested between ***momed*** and ***black*** (***momed*black*** in the results) and ***momed*** and ***Hispanic*** (***momed*Hispanic*** in the results). The overall interaction test (not shown)

TABLE 2.A ⬢ INTERACTION BETWEEN MOTHER'S EDUCATION AND RACE IN PREDICTING A CHILD'S EDUCATION

Parameter	Estimate	Standard Error	*t* Value	Pr > |t|
Intercept	8.384118921	0.19267909	43.51	<.0001
momed	0.163236396	0.01463782	11.15	<.0001
asvab	0.442150813	0.01054159	41.94	<.0001
black	1.122676462	0.29727422	3.78	0.0002
Hispanic	1.801221813	0.23440161	7.68	<.0001
momed*black	−0.068214539	0.02266732	−3.01	0.0026
momed*Hispanic	−0.147286595	0.01799430	−8.19	<.0001

was significant. Variables are defined in the same way as for question 4, chapter 1. That is:

- **momed:** The respondent's mother's education in years
- **Black, Hispanic:** dummy variables for these groups, relative to Whites.
- **Asvab:** This is the person's percentile rank on a national achievement test given in early high school. Here it is measured in 10% increases, so it varies from 0 to 10.

In the results, only ***Asvab*** was controlled, and its mean in this equation was 4.8.

Answer these questions:

a. What is the effect of mother's education on the child's education among Whites? No calculation is necessary here: The answer can be interpreted directly from the equation.
b. Use the results to calculate the effect of mothers' education on the child's education among Hispanics. Show the intercept and the slope among Hispanics.

3. The results for this question assess whether the effect of education on experience of discrimination varies by whether you are a visible minority, using the 2015 Canadian General Social Survey data.

   The dependent variable here is the number of institutions at which the respondent has experienced discrimination.

   The independent variables are

   - **educyrs**: Years of education
   - **vismin**: A dummy variable = 1 if a visible minority; 0 if not

In the regression results shown in Table 2.B (using PROC GLM) in SAS, there is a significant interaction between ***educyrs*** and the ***vismin*** dummy variable (***educyrs*vismin***).

TABLE 2.B ⬢ AN INTERACTION BETWEEN EDUCATION AND VISIBLE MINORITY STATUS IN PREDICTING THE EXPERIENCE OF DISCRIMINATION

Parameter	Estimate	Standard Error	*t* Value	Pr > |t|
Intercept	28.30707730	0.73756962	38.38	<.0001
educyrs	−0.20509793	0.05352192	−3.83	0.0001
vismin	6.13133315	0.84597840	7.25	<.0001
educyrs*vismin	−0.19211140	0.06220115	−3.09	0.0020

Answer these questions:

a. What is the effect (*the regression coefficient)* of education on reported discrimination among respondents who are in the nonvisible reference group? Just state the coefficient (the slope).
b. Use the results to calculate the effect of education on reported discrimination in visible minority groups.
c. Which group benefits more from education: visible minorities or others?

4. The results in Table 2.C focus on the relationship between child grades in school and their educational aspirations. In the regression results from SAS, there is a significant interaction between child grades and whether the mother has had depression problems earlier in life in predicting aspirations.

   The variables are:

   educaspirations: The dependent variable in the regression. It is the number of years of additional education the child intends to complete.
   cgrades: The child's report of their average grades in five subjects, on a scale from 1 (weak) to 5 (strong).
   momdepearly: A dummy variable = 1 if the mother had depression problems earlier in life, and =0 if not
   cgrades*momdepearly: the interaction between child grades and the mother's earlier depression problems.

TABLE 2.C ⬢ THE CONDITIONAL EFFECT OF CHILD GRADES ON EDUCATION ASPIRATIONS

Parameter	Estimate	Standard Error	*t* Value	Pr > |t|
Intercept	3.121025443	0.11687838	26.70	<.0001
cgrades	0.183210982	0.03205754	5.72	<.0001
momdepearly	0.462464158	0.21605413	2.14	0.0326
cgrades*momdepearly	−0.126095843	0.05963589	−2.11	0.0348

Answer these questions:

a. What is the effect of child grades on aspirations for children whose mothers did not have depression problems?
b. Use the results to calculate the effect of child grades on aspirations for children whose mothers *did* have depression problems.

5. Results are shown in table 2.D from a study of 9- to 16-year old children in husband-wife families in Toronto. The dependent variable here is an index of externalizing symptoms (aggression, hostility, and anger), and the main focus is the effect of maternal caring (from the Parental Bonding scale) on externalizing symptoms. Maternal caring measures the active support and nurturance of the mother as reported by the child.

   Model 1 is the additive model, Model 2 is the two-way interaction model, and Model 3 is the three-way interaction model. The independent variables in the output are

   - **momcare**: Maternal caring sub-scale from Parental Bonding
   - **female**: A dummy variable = 1 for female, 0 for male.
   - **teen**: A dummy variable = 1 for children 13-16, 0 for children 9–12
   - **femxteen**: female x teen
   - **mcarexfem**: momcare x female
   - **mcarexteen**: momcare x teen
   - **mcarexfemxteen**: momcare x female x teen

   a. Conduct a test to determine whether there is a three-way interaction between maternal care, child gender (female), and child age (teen).
   b. Whether it is significant or not, calculate the subgroup slopes for the effect of maternal caring in four groups: boys 9 to 12, boys 13 to 16, girls 9 to 12, and girls 13 to 16. In which group does maternal caring have the lowest impact on externalizing symptoms? (***Note:*** *You do not have to calculate intercepts in subgroups to answer this question).*
   c. Write a test statement to determine whether there is a difference in the effect of maternal caring for teenage boys versus teenage girls.

TABLE 2.D ⬢ THREE NESTED REGRESSION MODELS USED TO ESTIMATE A THREE-WAY INTERACTION

MODEL 1

Number of Observations Read	881
Number of Observations Used	878
Number of Observations with Missing Values	3

Weight: famweight weight by nativity, maternal employment, income, and kids 9 to 16

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	*F* Value	Pr > F
Model	3	94.52422	31.50807	35.26	<.0001
Error	874	780.92435	0.89351		
Corrected Total	877	875.44857			

(*Continued*)

TABLE 2.D ⬢ Continued

Root MSE	0.94525	R-Square	0.1080
Dependent Mean	0.01780	Adj R-Sq	0.1049
Coeff Var	5309.02523		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	*t* Value	Pr > |t|
Intercept	1	1.75446	0.20776	8.44	<.0001
momcare	1	−0.08379	0.00982	−8.54	<.0001
female	1	−0.20833	0.06412	−3.25	0.0012
teen	1	0.21181	0.06559	3.23	0.0013

MODEL 2

Number of Observations Read	881
Number of Observations Used	878
Number of Observations with Missing Values	3

Weight: famweight weight by nativity, maternal employment, income, and kids 9 to 16

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	*F* Value	Pr > F
Model	6	99.47322	16.57887	18.61	<.0001
Error	871	775.97534	0.89090		
Corrected Total	877	875.44857			

Root MSE	0.94388	R-Square	0.1136
Dependent Mean	0.01780	Adj R-Sq	0.1075
Coeff Var	5301.28200		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	1.53797	0.33632	4.57	<.0001
momcare	1	−0.07074	0.01614	−4.38	<.0001
female	1	−0.13807	0.41760	−0.33	0.7410
teen	1	0.47442	0.40677	1.17	0.2438
femxteen	1	0.27475	0.13172	2.09	0.0373
mcarexfem	1	−0.00879	0.01965	−0.45	0.6549
mcarexteen	1	−0.01977	0.01976	−1.00	0.3174

MODEL 3

Number of Observations Read	881
Number of Observations Used	878
Number of Observations with Missing Values	3

Weight: famweight weight by nativity, maternal employment, income, and kids 9 to 16

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	*F* Value	Pr > F
Model	7	102.92051	14.70293	16.56	<.0001
Error	870	772.52806	0.88796		
Corrected Total	877	875.44857			

Root MSE	0.94232	R-Square	0.1176
Dependent Mean	0.01780	Adj R-Sq	0.1105
Coeff Var	5292.53244		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	*t* Value	Pr > |t|
Intercept	1	1.20057	0.37691	3.19	0.0015
momcare	1	−0.05431	0.01814	−2.99	0.0028
female	1	0.58582	0.55569	1.05	0.2921
teen	1	1.27207	0.57341	2.22	0.0268
femxteen	1	−1.30881	0.81439	−1.61	0.1084
mcarexfem	1	−0.04354	0.02639	−1.65	0.0992
mcarexteen	1	−0.05959	0.02824	−2.11	0.0352
mcarexfemxteen	1	0.07776	0.03947	1.97	0.0491

6. The results that follow (Table 2.E) are from an analysis using data from the National Survey of Families and Households at Waves 1 and 2. This analysis considers the impact of marital problems reported at Wave 1 on the impact of divorce on depression between Waves 1 and 2 in a sample of married respondents at Wave 1.

   The variables are
   - **cesd2**: Depression at Wave 2
   - **cesd1**: Depression at Wave 1
   - **div12**: = 1 if the respondent got divorced between Waves 1 and 2
   - = 0 if the respondent stayed married
   - **marprob1**: An index of marital problems reported at Wave 1
   - **marprobxdiv:** = div12*marprob1 (an interaction)

   The displayed output includes descriptive statistics, an additive model showing the effect of divorce on depression at Wave 2 controlling for prior marital problems and depression at Wave 1, and an interactive model.

   Answer these questions:

   a. Conduct a test *or cite evidence in the output* concerning the significance ($p < .05$) of the interaction between divorce and prior marital problems.
   b. Assuming that there is a significant interaction and using the information in the descriptive statistics about the mean and standard deviations of variables in the model, calculate (only) the effect of divorce at +1 and -1 standard deviations from the mean level of marital problems.

TABLE 2.E ● TESTING AN INTERACTION BETWEEN PRIOR MARITAL PROBLEMS AND THE EFFECT OF DIVORCE.

The REG Procedure

Model: MODEL1

Dependent Variable: cesd2

Number of Observations Read	5456
Number of Observations Used	5157
Number of Observations with Missing Values	299

Descriptive Statistics

Variable	Sum	Mean	Uncorrected SS	Variance	Standard Deviation
Intercept	5853.47443	1.00000	5853.47443	0	0
div12	598.18626	0.10219	598.18626	0.10416	0.32274
marprob1	10878	1.85841	24127	0.75850	0.87092
cesd1	6003.51033	1.02563	15399	1.79241	1.33881
cesd2	6102.47533	1.04254	15303	1.73413	1.31686
marprobxdiv	1455.88158	0.24872	4264.68370	0.75690	0.87000

Weight: MUFINW93 The person weight for NSFH2 main respond

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	1351.23858	450.41286	305.80	<.0001
Error	5153	7589.92205	1.47291		
Corrected Total	5156	8941.16063			

Root MSE	1.21364	R-Square	0.1511
Dependent Mean	1.04254	Adj R-Sq	0.1506
Coeff Var	116.41161		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	0.46271	0.03980	11.63	<.0001
div12	1	0.37851	0.05394	7.02	<.0001
marprob1	1	0.10501	0.02051	5.12	<.0001
cesd1	1	0.33736	0.01301	25.93	<.0001

MODEL 2

regression ces-d on marital situation 74

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The REG Procedure

Model: MODEL2

Dependent Variable: cesd2

Number of Observations Read	5456
Number of Observations Used	5157
Number of Observations with Missing Values	299

Weight: MUFINW93 The person weight for NSFH2 main respond

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	*F* Value	Pr > F
Model	4	1358.21235	339.55309	230.70	<.0001
Error	5152	7582.94828	1.47185		
Corrected Total	5156	8941.16063			

Root MSE	1.21320	R-Square	0.1519
Dependent Mean	1.04254	Adj R-Sq	0.1512
Coeff Var	116.36940		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	*t* Value	Pr > |t|
Intercept	1	0.42442	0.04350	9.76	<.0001
div12	1	0.63165	0.12819	4.93	<.0001
marprob1	1	0.12656	0.02277	5.56	<.0001
cesd1	1	0.33700	0.01301	25.91	<.0001
marprobxdiv	1	-0.10963	0.05037	-2.18	0.0295

7. Results in this question (Table 2.F) are from a model estimating a three-way interaction between work–family conflict, gender, and perception of neighborhood disorder in predicting distress, using the 2009–2011 Toronto Study on Neighbourhood Effects on Health and Well-Being (O'Campo et al., 2015). The three-way term is significant, so you should assume there ***is*** a three-way interaction. There are two controls as well—education and marital status—but they are ***not*** relevant in this question.

   The means and standard deviations for the variables that follow are part of the output. The independent variables in the output are

   - **wfc**: A measure of work-family conflict
   - **FEMALE**: A dummy variable = 1 for female, 0 for male
   - **neighdisorder**: The respondent's perception of disorder in the neighborhoodenvironment, including the presence of trash, litter, loud noise, heavy traffic, gang activity, crime, and drug dealers
   - **reduc**: Years of education
   - **married**: A dummy variable for married =1 if married, 0 if not.
   - **neighdisorderxwfc**: neighdisorder x wfc
   - **neighdisorderxfemale**: neighdisorder x female
   - **wfcxfemale**: wfc x female
   - **neighdisorderxwfcxfemale**: neighdisorder x wfc x female

   a. Use the output for the descriptive statistics to figure out the levels of neighborhood disorder corresponding to +1 and -1 standard deviations from the mean.

   b. ***ONLY*** figure out the slopes in this question. Calculate the slope for the effect of work–family conflict on distress among women in neighborhoods with high disorder (+1 SD above the mean) and the slope for work–family conflict among men in neighborhoods with high disorder (+1 SD below the mean).

   c. Write out a TEST statement that tests the significance of the slope for the effect of work–family conflict among women in high disorder (+1 SD) neighborhoods.

TABLE 2.F ⬢ A THREE-WAY INTERACTION BETWEEN NEIGHBORHOOD DISORDER, WORK-FAMILY CONFLICT, AND GENDER

The MEANS Procedure

Variable	Label	N	Mean	Sid Dev	Minimum	Maximum
neighdisorder		1702	3.9124559	1.6565754	2.0000000	10.0000000
wfc		1702	8.9994125	3.2483885	4.0000000	16.0000000
FEMALE	Participant is female	1702	0.5329025	0.4990629	0	1.0000000
reduc		1702	7.9747356	0.3123121	1.0000000	8.0000000
married		1702	0.5564042	0.4969544	0	1.0000000

The REG Procedure
Model: MODEL1
Dependent Variable: distress

Number of Observations Read	1702
Number of Observations Used	1702

Weight: nehwweight weight by gender, nativity, hhincome, and household size

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	*F* Value	Pr > F
Model	9	21448	2383.14004	41.60	<.0001
Error	1692	96940	57.29343		
Corrected Total	1701	118389			

Root M SE	7.56924	R-Square	0.1812
Dependent Mean	10.73710	Adj R-Sq	0.1768
Coeff Var	70.49618		

Parameter Estimates

Variable	Label	DF	Parameter Estimate	Standard Error	t Value	Pr > Iti
Intercept	Intercept	1	7.31462	5.00171	1.46	0.1438
neighdisorder		1	−0.05368	0.45601	−0.12	0.9063
wfc		1	0.69149	0.20710	3.34	0.0009
FEMALE	Participant is female	1	5.43195	2.68617	2.02	0.0433
reduc		1	−0.34766	0.57854	−0.60	0.5480
married		1	−2.73686	0.39920	−6.86	<.0001
neighdisorderxwfc		1	0.03551	0.04952	0.72	0.4734
neighdisorderxfemale		1	−0.79922	0.62730	−1.27	0.2028
wfcxfemale		1	−0.89026	0.28612	−3.11	0.0019
neighdisorderxwfcxfemale		1	0.18287	0.06537	2.80	0.0052

8. The effect of divorce may dissipate with time—but differentially across gender. The results for this question in Table 2.G test a two-way interaction between gender and time since divorce, considered as a set of dummy variables, in predicting depression at Wave II of the NSFH. Both variables are therefore categorical in the equation, with the stably married the reference group for time since divorce.

   The attached output includes only the interaction model and a post-hoc test for any interaction between time since divorce and gender. The variables are as follows:

   - **divlast2**: The experience of divorce in the last two years before Wave 2.
   - **div2to4**: Divorce 2 to 4 years before Wave 2
   - **div4to6**: Divorce 4 to 6 years before Wave 2
   - **violence**: A count of the number of violent incidents in the marriage per year at Wave 1
   - **cesd2tot**: A depression scale at Wave 2
   - **cesd1tot**: The same depression scale at Wave 1
   - **female**: A dummy variable for female (= 1 if female, 0 if male).
   - **femxdivlt2**: female x divlast2
   - **femxdiv24**: female x div2to4
   - **femxdiv46**: female x div4to6

   a. What result in the output provides evidence that there is a two-way interaction between gender and time since divorce?
   b. Calculate the slope for the effect of female on depression among those:
      1. still married
      2. divorced in the last two years
      3. divorced 4 to 6 years ago
   c. Write out a TEST statement that tests the significance of the difference in the effect of divorce for women divorced 4 to 6 years ago versus 2 to 4 years ago, using the variable names in the equation.

TABLE 2.G ⬢ GENDER SPECIFIC EFFECTS OF DIVORCE BY TIME SINCE DIVORCE

Number of Observations Read	5213
Number of Observations Used	5028
Number of Observations with Missing Values	185

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	9	207660	23073	121.76	<.0001
Error	5018	950871	189.49212		
Corrected Total	5027	1158532			

Root MSE	13.76561	R-Square	0.1792
Dependent Mean	13.05564	Adj R-Sq	0.1778
Coeff Var	105.43806		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	6.28216	0.34066	18.44	<.0001
divlast2	1	7.12725	1.57191	4.53	<.0001
div2to4	1	3.92742	1.52619	2.57	0.0101
div4to6	1	−0.91914	1.33123	−0.69	0.4899
violence	1	1.96667	0.29385	6.69	<.0001
female	1	2.28131	0.41985	5.43	<.0001
cesd1tot	1	0.35242	0.01323	26.63	<.0001
femxdivlt2	1	1.50082	2.09839	0.72	0.4745
femxdiv24	1	2.72668	1.97904	1.38	0.1683
femxdiv46	1	4.88633	1.82143	2.68	0.0073

(*Continued*)

TABLE 2.G ● Continued

Test femxdiv Results for Dependent Variable cesd2tot				
Source	DF	Mean Square	F Value	Pr > F
Numerator	3	571.95487	3.02	0.0287
Denominator	5018	189.49212		

Appendix

We present syntax below showing how to code the two-way interaction example in this chapter in SAS and STATA. The purpose here is not to teach the basics of coding in each program but to broadly outline the differences in the organization and logic of coding.

The syntax below is annotated using comments in the coding. We encourage this in both programs. In data analysis, you have to leave a trail of evidence about what you have done in order to reproduce findings and/or make revisions to finished papers.

SAS Code to Create the Data for Running the Two-Way Interaction

```
/*1*/

/* Comments start and end with these delimiters */


/* This is a data step in SAS. This creates the new data you will eventually analyze in a
PROC step, using the raw data as input. The data step is used to create new variables for
analysis, recode variables, subset the sample, or whatever you need to do to fine tune your
variables for analysis.*/


/* Start with a DATA statement. Give a name to a temporary data set you will create for
this run only. All SAS statements end in a semi-colon */


/* The SET statement is a statement that tells SAS to read an existing data set. It has a two-
level name: the first level is a special library name which you have defined telling SAS in which
folder the data resides on your computer. After the "dot", the second level is the file name of
the data in that folder. SAS uses the parenthesis to introduce options: here "keep=" tells SAS
which variables to read in from the data set. This is mainly useful very large data sets.*/


data temp;
       set nsfhdata.nsfh1(keep=mcaseid cmint m484 m540a m540b m535--m538 m2bp01 irwage
       irearn irtot1 ihtot1 m530t01m m532t02m m532t03m m532t04m educat weight
          m532t05m m532t06m m532t07m m532t08m m532t09m m532t10m m529t01m m531t02m m531t03m
          m531t04m m531t05m m531t06m m531t07m m531t08m m531t09m m531t10m  m534t01 m534t02
          m534t03 m534t04 m534t05 m534t06 m534t07 m534t08 m534t09 m534t10);


    /* 2. This is how you subset data. Here we select people who identify as Black, White,
or Hispanic in the wave 1 NSFH data. This also excludes people missing on this variable
(codes 97 and above). The IF statement simply states a condition for reading the data. */


     if 1<=m484<=6;
```

```
/*3. Renaming and recoding variables.First use IF- THEN statements to recode missing values to system missing in SAS ("."). Then use "newvar= function(oldvar)" type statements to create new variables. The round function divides income in dollars by 1000 and rounds it to one decimal place. You can also use "newvar = oldvar" statements just to rename variables*/.

    if irtot1>=999996 then irtot1=.;
    if ihtot1>=99999996 then ihtot1=.;
    if irwage>=999996 then irwage=.;
    if irearn>=999997 then irearn=.;

    if educat>=90 then educat=.;
    educ=educat;

    if m2bp01>95 then  m2bp01=.;
    age = m2bp01;

    SEI=round((m540b/100),1);
    if sei>=99 then sei=.;

    rtotinc=round(irtot1/1000,1);
    hhinc=round(ihtot1/1000,1);
    rjobinc=round(irwage/1000,1);
    rearninc=round(irearn/1000,1);

    /* 4. Using an array to figure out total years in the current job to measure seniority. Arrays are just lists of variables you can refer to with a single label, named by the ARRAY statement.

    The DO loop performs the same action on each element of the array in turn.

    Here we are looking for the first job in a job history that is full-time and still
    ongoing (endwrk(i)=9995). When this happens in the list, a new variable labeled
    "cmstrtjob" is created. This variable is the century month of the first month of the
    current job.

The IF / THEN statement tells the DO loop to leave -- to quit -- when the new     variable takes on a real value, denoted by any value greater than "."*/

    array startwrk(10) m529t01m m531t02m m531t03m m531t04m m531t05m m531t06m m531t07m
          m531t08m m531t09m m531t10m;
    array endwrk(10) m530t01m m532t02m m532t03m m532t04m m532t05m m532t06m m532t07m
          m532t08m m532t09m m532t10m;
    array full(10) m534t01 m534t02 m534t03 m534t04 m534t05 m534t06 m534t07 m534t08
          m534t09 m534t10;

    do i=1 to 10;
       if 0<startwrk(i)<9990 and full(i)=1 and endwrk(i)=9995 then cmstrtjob=startwrk(i);
       if cmstrtjob>. then leave;
    end;
```

```
     /* 5. This creates the seniority variable "yrscurrjob" by subtracting the starting
century month of the current job from the century month of the interview ("cmint"), and
dividing by 12 to turn the result into years. Total time in the labor force is also created
here, by taking the difference between the current month and the starting century month of
the first job.*/

     yrscurrjob=(cmint-cmstrtjob)/12;
     yrslabor=(cmint-m529t01m)/12;

     /* 6. This deletes observations within a certain range of
           occupational categories on the current occupation variable.
           This was eventually deleted from the program using a single line comment (*) */

     *if 473<=m540a<=499 then delete;

     /* 7. Racial dummy variables -- white is the invisible reference
           Use IF/THEN/ELSE statements to create the dummy variables.
           The target group is coded "1", all other groups are coded "0"
           by the ELSE statement.*/

     if m484=1 then black=1; else black=0;
     if 3<=m484<=6 then hisp=1; else hisp=0;

     /* 8. Interactions created here. The "*" multiplies already created variables */

     blackxeduc=black*educat;
     hispxeduc=hisp*educat;
     agexeduc =age*educat;
run;
```

Matching STATA Code

```
*In STATA, comments start and end with asterisks only. 'Enter' serves as a delimiter for
executable statements --compared to SAS, which uses a semi-colon*
*If working in a syntax file (which is referred to as a 'do-file' in STATA) and wish to
continue a command line, use ///*
*to let STATA know you are not done with the command yet*
*Compared to SAS, STATA --by default--transforms all cases in the data after each command.*
*This is unique compared to SAS, which executes all commands used to transform data one
case at a time. The unique 'vertical*horizontal' vs. 'horizontal*vertical' treatment of the
data, and focus on one versus all cases, distinguishes the two programs*

*STATA is also unique in that you can execute line commands to make permanent changes to
the dataset. People often use the 'command' window to execute statements step by step*
*This could be done in SAS as well, but SAS usually is set up to produce a new data set
in its DATA step, much like a batch file-based program. In SAS, users are often creating
temporary data files for use in the current analysis, preserving the original data*
*This is not the case in STATA. Unless you set up the do-file in a*
```

```
*batch file-based approach and save the data as a secondary file, the commands you execute will alter your original variables, which can become a problem*

*We therefore suggest to use such an approach when creating your do-file. See the example, below*


*First, open the permanent dataset*


use "C:{insert path to data here}.dta", clear


*1. Keep the variables you wish to use in the analyses using the statement 'keep'*

keep mcaseid cmint m484 m540a m540b m535--m538 m2bp01 irwage irearn irtot1 ///
ihtot1 m530t01m m532t02m m532t03m m532t04m educat weight ///

m532t05m m532t06m m532t07m m532t08m m532t09m m532t10m m529t01m m531t02m m531t03m ///
m531t04m m531t05m m531t06m m531t07m m531t08m m531t09m ///

m531t10m  m534t01 m534t02 m534t03 m534t04 m534t05 m534t06 m534t07 m534t08 m534t09 ///
m534t10


*2. keep the subsample you want using a 'keep if' statment*

keep if m484==1/6



*3. renaming and recoding the individual income variables. STATA uses 'replace' 'if' statements. Missing variables are denoted as '.'*

replace irtot1=. if irtot1>=999996

replace ihtot1=. if ihtot1>=99999996

replace irwage=. if irwage>=999996

replace irearn=. if irearn>=999997



*The following rounds the variables of interest, similar to SAS, you use a 'round' option. Note here, the 'gen' statement - short for generate, which produces a new variable. As you will see below, you can reduce this further to referencing 'g' only*

gen rtotinc=round((irtot1/1000),1)

gen  hhinc=round((ihtot1/1000),1)

gen rjobinc=round((irwage/1000),1)

gen rearninc=round((irearn/1000),1)


replace educat=. if educat>=90

g educ=educat


replace m2bp01=. if m2bp01>=95

g age = m2bp01

g sei=round((m540b/100),1)

replace sei=. if sei>=99
```

```
*4. using a 'loop' to generate a new set of variables for cm start job*

foreach var of varlist m529t01m m531t02m m531t03m m531t04m m531t05m m531t06m /// m531t07m
m531t08m m531t09m m531t10m {
     g cmstrtjob`var'= 'var' if foreach 'x' local endwrk
     }
     ]

*compared to SAS, three-variable based arrays are difficult to code in STATA - the
following line commands are more common for this type of variable transformation*

gen cmstrtjob=.

replace cmstrtjob=m529t01m if ((m529t01m==1/9990) & m534t01==1 & m530t01m==9995))
replace cmstrtjob=m531t02m if ((m529t02m==1/9990) & m534t02==1 & m530t02m==9995))
replace cmstrtjob=m531t03m if ((m529t03m==1/9990) & m534t03==1 & m530t03m==9995))
replace cmstrtjob=m531t04m if ((m529t04m==1/9990) & m534t04==1 & m530t04m==9995))
replace cmstrtjob=m529t05m if ((m529t05m==1/9990) & m534t05==1 & m530t05m==9995))
replace cmstrtjob=m529t06m if ((m529t06m==1/9990) & m534t06==1 & m530t06m==9995))
replace cmstrtjob=m529t07m if ((m529t07m==1/9990) & m534t07==1 & m530t07m==9995))
replace cmstrtjob=m529t08m if ((m529t08m==1/9990) & m534t08==1 & m530t08m==9995))
replace cmstrtjob=m529t09m if ((m529t09m==1/9990) & m534t09==1 & m530t09m==9995))
replace cmstrtjob=m529t10m if ((m529t10m==1/9990) & m534t10==1 & m530t10m==9995))

*5. The following creates the same variables as in the SAS program for 'yrscurrjob' and
'yrslabor'*
g yrscurrjob=(cmint-cmstrtjob)/12
g yrslabor=(cmint-m529t01m)/12

*6. We create the racial dummy variables here. Note, the unique approach to recoding and
generating new variables in the same command line*
recode m484 1=1 2/6=0, g (black)
recode m484 1/2=0 3/6=1, g (hisp)

*7. Similar to the SAS program, we generate education interactions with race by multiplying
the variables together using an asterisk*

g blackxeduc=black*educat
g hispxeduc=hisp*educat
g agexeduc =age*educat

*8. save a new dataset with the changes*
save "C:{insert path to data here}.dta", replace
```